1958

Dynamic linear programming of conservation alternatives, including household consumption

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DYNAMIC LINEAR PROGRAMMING
OF CONSERVATION ALTERNATIVES, INCLUDING
HOUSEHOLD CONSUMPTION

by

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A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Agricultural Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

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Signature was redacted for privacy.

Dean of Graduate College

Iowa State College
1958

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I. INTRODUCTION

A. Description of the Study Area

This study is located in the Ida-Monona soil association area of western Iowa.[1] These strongly rolling loessial soils were originally fertile, and are productive when managed in an efficient manner. However, their productivity has been progressively diminished by erosion. Loss of fertile topsoil through sheet erosion has been particularly great in some areas. Gully erosion is serious because of the vertical structure of the Ida and Monona soils. Some gullies, now over 100 feet deep, cut back several hundred feet each year. Consequently, roads, bridges, fences and farm buildings must be relocated frequently. More serious than the deep gullies are the small gullies and depressions which develop in cultivated fields. It is estimated that the annual loss of soil in the area averages about 20 tons per acre.[2] On some farms it is as high as 60 tons, an amount equivalent to nearly one-half

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[1]For a more detailed description of the study area see: Ross V. Baumann, Earl O. Heady and Andrew R. Aandahl. Costs and returns for soil-conserving systems of farming on Ida-Monona soils in Iowa. Iowa Agr. Exp. Sta. Res. Bul. 429. 1955; A. Gordon Ball, Earl O. Heady and Ross V. Baumann. Economic evaluation of use of soil conservation and improvement practices in western Iowa. U.S.D.A. Tech. Bul. No. 1162. 1957; S.M. Aijaz Husain. Optimum resource allocation for erosion control farming on Ida-Monona soils. Unpublished Ph.D. Thesis, Ames, Iowa, Iowa State College Library. 1957.

[2]Frey, John C. Some obstacles to soil erosion control in western Iowa. Iowa Agr. Exp. Sta. Res. Bul. 391. 1952.

inch of topsoil.

Farm practices common in the area intensify soil losses by erosion. Corn is the main crop grown. While many are operated on a cash grain basis, farms generally have livestock enterprises organized around the corn and oats produced. A large percentage of the farms do not follow an established rotation. When rotations do exist, they commonly include two years of corn, one of small grain and one of hay. Even then, row crops typically are planted up and down hills with slopes that often exceed 15 percent.

Various soil conservation practices such as contouring and terracing, contour strip-cropping, sodded waterways, improved rotations, and permanent seeding of steep land are needed on most farms in the soil area. Such practices would help conserve soil resources, reduce damage from floods, and help to maintain or augment the low farm incomes through time.

B. Setting for the Study

Several economic studies on soil conservation practices in the Ida-Monona soil area have been made.[3] However, these studies do not specify the transition adjustments over time that a farmer must make in adopting a final conservation plan. In other words, a series of intermediate plans, as well as the final conservation plan, should be specified. In this study

[3]Baumann, Heady and Aandahl, op. cit., p. 1.
Ball, Heady and Baumann, op. cit., p. 1.
Husain, op. cit., p. 1.

a series of yearly plans covering a five year period are presented. The plan for each year is the best possible plan in terms of the five year optimum. The plans show the farmer how to make the necessary adjustments over a five year period towards the final conservation plan.

Under actual farm conditions, the length of time required for a farmer to adopt the final conservation plan will be a function of the resources available. The period required will vary with the productivity of the land, the farmer's capital and equity position, managerial ability and labor supply. Additionally, the time needed for the adoption of a conservation program will depend upon the relative marginal return to capital management and labor resources invested in conservation practices as compared to the return on the same capital invested in non-conservation practices.

The dynamic linear programming models developed in this study permit t years of activities and restrictions to be programmed including the year of the final conservation plan. However, due to the restrictions of the IBM 650 "library" program, in computing the plans, only 5 years of restrictions and activities could be programmed. Therefore, the plans presented do not necessarily represent the final conservation plan. They only show him the adjustments to make during the first five years in moving toward the final conservation plan.

Plans are presented for two farm sizes based on different conservation alternatives. These alternatives include:

(1) <u>no</u> crop fertilization <u>nor</u> terracing and contouring of cropland; (2) crop fertilization <u>but</u> <u>no</u> terracing and contouring of cropland; (3) crop fertilization <u>and</u> terracing and contouring of cropland.

In each conservation alternative studied, the cost of family living (household consumption) is considered. A consumption activity is necessary, because it will take preference over farm production in the allocation of available capital. In this study, an average or "typical" family living expenditure is deducted from the capital supply of each of the five years.*

Each of the optimum five-year plans applies to specific farms. However, the plans can be used as guides for adoption of soil conservation practices on similar farms in the Ida-Monona soil area.

The limitations of this study are: (1) Final or optimal conservation plans are not necessarily included in the plans presented. (2) Only three conservation alternatives (<u>i</u>. <u>e</u>. combinations of conservation practices) are studied. (3) Only one price level is used for all plans. (4) Many more crop and livestock enterprises could be included in the production possibilities. (5) No attempt has been made to incorporate soil bank payments and resulting land use programs.

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*In Situation I, (see p. 33), family living is only deducted from the capital supply of four years.

C. Technique and Method of Analysis

This study employs two dynamic linear programming models in determining conservation plans for a period of five years. The technique represents an improvement over static analysis of conservation adjustment in that plans are specified for each year of the five-year period. Specifically, the method can be used to determine the most profitable farming plan for each year under each conservation situation studied.

II. OBJECTIVES

The general objective of this study is to develop a method of dynamic linear programming applicable for determining optimum farm plans for a five-year period. The farm situations studied are on the Ida-Monona soil association of western Iowa. The specific objectives are:

1. To develop a method of dynamic linear programming that permits simultaneous programming of t years of activities and restrictions.
2. To determine optimum five-year plans for a 160- and 280-acre farm employing alternative levels of conservation.
3. To determine, for each of the five years, the optimum crop and livestock plan at different levels of conservation, after first taking into consideration farm household consumption.
4. To determine the effects of household consumption (family living) on conservation plans and present and future incomes.
5. To interpret the results of the dynamic linear programs in terms of their implications to conservation recommendations to farmers, conservation and non-conservation investment opportunities and, cropping and livestock opportunities.

III. TECHNIQUE AND METHODOLOGY OF DYNAMIC LINEAR PROGRAMMING

Two models for dynamic linear programming are presented in this study. The first model, to be referred to as the expansion model, treats individual crops and non-crop enterprises as activities. The second model, to be referred to as the rotation model, treats crop rotations and single (or individual) non-crop enterprises as activities. The latter model reduces the number of restrictions necessary. Both models involve time and have similar computational procedures and algebraic structure. The models differ only in the treatment of crop activities, and hence, in the form of the input-output matrix.

In this chapter, the following procedure is used: First, a static linear programming model is described as a basis for illustrating dynamic linear programming. Then, the dynamic linear programming models are explained.

A. Static Linear Programming Model

Numerous empirical studies using static linear programming have been made. Hence, since the details of the static model are outlined elsewhere, only a brief outline is presented here.[4] The following equations summarize the

[4] For other examples of the technique of static linear programming see: Heady, Earl O. Simplified presentation and logical aspects of linear programming technique. Journal of Farm Economics, Volume XXXVI, No. 5: (Continued on next page)

static linear programming model.

Let $b_1, b_2, \ldots, b_i, \ldots, b_m$ represent the resource restrictions (or supplies) and, $x_1, x_2, \ldots, x_j, \ldots, x_n$ be the unknown activity levels. Also, let $a_{11}, a_{12}, \ldots, a_{ij}, \ldots, a_{mn}$ represent the coefficients (input-output or transformation rates) of the unknown x_j's and, $c_1, c_2, \ldots, c_j, \ldots, c_n$ be the net revenue per unit of activity. Furthermore, let X represent a column vector of x_j's, c a column vector of c_j's and, B a column vector of b_i's.[5] Also, let A represent a matrix of a_{ij}'s where the matrix A includes the elements a_{ij}, $i = 1, \ldots, m$, $j = 1, \ldots, n$. That is,

$$A = \begin{vmatrix} a_{11}, a_{12}, \ldots\ldots\ldots\ldots, a_{1n} \\ a_{21}, a_{22}, \ldots\ldots\ldots\ldots, a_{2n} \\ \vdots \\ a_{m1}, a_{m2}, \ldots\ldots\ldots\ldots, a_{mn} \end{vmatrix}$$

Then, the optimum plan is the one that allows the condition of equation (1).

(Continued from previous page)
1035-1048. December, 1954; Gilson, James C. Optimum livestock production under varying resource and price-cost situations in northeastern Iowa - an application of linear programming. Unpublished Ph.D. Thesis. Ames, Iowa, Iowa State College Library. 1954; Charnes, A., Cooper, W. W. and Henderson, A. An introduction to linear programming. New York, John Wiley and Sons. 1953

[5]See: Charnes, Cooper and Henderson, ibid., p. 8.

$$\text{maximize } f(X) = Z = c'X \tag{1}$$

$$\text{subject to } AX \leq B \qquad x \geq 0$$

$$\text{where } x = 1,2,\ldots\ldots,n$$

$$c = 1,2,\ldots\ldots,n$$

$$b = 1,2,\ldots\ldots,m$$

and c' denotes the transpose of c. The problem is to maximize the net profit, $f(X)$, by solving for the unknown x_j's.

In terms of the above matrix notation, the static linear programming model may be written as shown in equations (2) where a_{ij}, x_j, c_j and b_i are as defined previously.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\leq b_m \end{aligned} \tag{2}$$

$$\text{where } x_j \geq 0$$

and

$$f(X) = Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \tag{3}$$

$$\text{or } Z = \sum_{j=1}^{n} x_jc_j$$

is a maximum.

To make the problem easier, "slack" or "disposal" variables are introduced, and the inequalities of equations (2) are replaced with equalities in equations (4). The variable x_{n+i} is a "slack" variable because it accounts for the excess of the right-hand side of equations (2) over the left-hand side. The equations thus become:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + a_{1n+1}x_{n+1} &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + a_{2n+2}x_{n+2} &= b_2 \qquad (4) \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + a_{mn+m}x_{n+m} &= b_m
\end{aligned}
$$

where

$$
\begin{aligned}
x_1 &\geqslant 0 \\
&\vdots \\
x_n &\geqslant 0 \\
x_{n+1} &\geqslant 0 \qquad (5) \\
&\vdots \\
x_{n+m} &\geqslant 0
\end{aligned}
$$

and $$f(X) = Z = c_1x_1 + c_2x_2 + \dots\dots + c_{n+m}x_{n+m} \qquad (6)$$

is a maximum. Equations (5) shows that an activity cannot be carried on at a negative level.

The "slack" vectors thus change the inequalities of equations (2) into equalities in equations (4) at a cost of introducing m additional non-negative unknowns. The elements of the "slack" vectors are in the form of

$$
\begin{aligned}
& a_{in+1} = 1 && i = 1, \dots m \\
\text{where}\quad & a_{in+j} = 0 && i, j = 1, \dots m;\ i \neq j
\end{aligned} \qquad (7)
$$

which is an identity matrix. Also, equations (2) and (4) indicate that consumption or use of each resource (b_i), for all activities (x), must not exceed the quantity of each available resource (b_i).

B. Dynamic Linear Programming

In contrast to static linear programming, where usually a single year is considered, the technique of dynamic linear programming provides a single optimum program for a period of t years. In dynamic programming, the plan of each year is the most profitable in terms of the t year optimum. Hence, the t years are drawn together to form an interrelated plan. In this study, optimum plans are presented for a five-year period where the plan for each year is the most profitable in terms of the five-year optimum. The dynamic linear programming models used in this study are the expansion and rotation models.

C. Dynamic Linear Programming - Expansion Model

1. Logic and technique of the expansion model

The logic and technique of the expansion model can be explained by a simple three-year programming example. Consider the following problem: How much corn (C), oats (O), hay (M) and hogs should be produced each year to maximize profits over a three-year period. The "fixed" resources are land, capital and June-October labor. (For simplicity, assume the other months of the year are not restrictive for crop and livestock production). An annual consumption activity, family living (household consumption), is also necessary because not all revenue (i.e. capital) obtained from each year's crop and hog production will be available for farm production the following year. A certain amount of capital will be needed for household consumption. Two other activities, grain buying and capital transfer, are also included in each year's enterprises. Grain buying allows the purchase of extra corn for livestock feed when necessary. Capital transfer allows surplus capital of one year to be utilized the following year.

To keep the explanation as simple as possible, the following cropping restrictions are assumed: only corn may follow corn, oats follow oats and hay follow hay. The net revenues and resource requirements of each activity are assumed to be calculated on a per unit basis.

In the expansion model, because more than one year is programmed at one time, additional subscripts are required to denote the year in which the activity or restriction occurs. Denote the year of the program by the subscript k, where $k = 1, 2, \ldots, k, \ldots, t$. In the example presented (see Table 1), $t = 3$. Also, denote the number of the row (or restriction) by i, where $i = 1, 2, \ldots, i, \ldots, n$; and the number of the column (or activity) by j, where $j = 1, 2, \ldots, j, \ldots, m$. Thus, element a_{ijk} is the requirement of the j^{th} activity for resource i in year k; b_{ik} the amount of restriction i in year k; c_{jk} the net revenue of the j^{th} activity in

Table 1. Expansion model of dynamic linear programming. Original input-output matrix for

Resource used or crop and livestock produced each year	Letter to identify	Supply, remainder or output	Year 1						
			$c_{1,1}$	$c_{2,1}$	$c_{3,1}$	$c_{4,1}$	$c_{5,1}$	$c_{6,1}$	$c_7 =$
			C_1	O_1	M_1	Hogs$_1$	Grain buying$_1$	Capital transfer$_1$	Fami: livi
		P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Yr. 1. Capital	P_{19}	$b_{1,1}$	$a_{1,1,1}$	$a_{1,2,1}$	$a_{1,3,1}$	$a_{1,4,1}$	$a_{1,5,1}$	$a_{1,6,1}$	$a_{1,7,}$
Yr. 1. Land	P_{20}	$b_{2,1}$	$a_{2,1,1}$	$a_{2,2,1}$	$a_{2,3,1}$				
Yr. 1. June-Oct. labor	P_{21}	$b_{3,1}$	$a_{3,1,1}$	$a_{3,2,1}$	$a_{3,3,1}$	$a_{3,4,1}$			
Yr. 1. Feed grain	P_{22}	$b_{4,1}$	$-a_{4,1,1}$	$-a_{4,2,1}$		$a_{4,4,1}$	$-a_{4,5,1}$		
Yr. 1. Forage feed	P_{23}	$b_{5,1}$			$-a_{5,3,1}$	$a_{5,4,1}$			
Family living	P_{24}	b_6							$a_{6,7,}$
Yr. 2. Capital	P_{25}	$b_{7,2}$	$-a_{7,1,2}$	$-a_{7,2,2}$	$-a_{7,3,2}$	$-a_{7,4,2}$	$-a_{7,5,2}$	$-a_{7,6,2}$	$a_{7,7,}$
Yr. 1. C_1 land	P_{26}	$b_{8,2}$	$-a_{8,1,2}$						
Yr. 1. O_1 land	P_{27}	$b_{9,2}$		$-a_{9,2,2}$					
Yr. 1. M_1 land	P_{28}	$b_{10,2}$			$-a_{10,3,2}$				
Yr. 2. June-Oct. labor	P_{29}	$b_{11,2}$							
Yr. 2. Feed grain	P_{30}	$b_{12,2}$							
Yr. 2. Forage feed	P_{31}	$b_{13,2}$			$-a_{13,3,2}$	$a_{13,4,2}$			
Yr. 3. Capital	P_{32}	$b_{14,3}$							$a_{14,7,}$
Yr. 2. C_2 land	P_{33}	$b_{15,3}$							
Yr. 2. O_2 land	P_{34}	$b_{16,3}$							
Yr. 2. M_2 land	P_{35}	$b_{17,3}$							
Yr. 3. June-Oct. labor	P_{36}	$b_{18,3}$							
Yr. 3. Feed grain	P_{37}	$b_{19,3}$							
Yr. 3. Forage feed	P_{38}	$b_{20,3}$							
Opportunity cost	z_{jk}	0	$z_{1,1} = 0$	$z_{2,1} = 0$..					
Marginal revenue	$z_{jk}-c_{jk}$	0	$z_{1,1}-c_{1,1}$	$z_{2,1}-c_{2,1}$..					

utput matrix for a 3-year program

					Year 2				
1	$c_{6,1}$	$c_7 = -M$	$c_{8,2}$	$c_{9,2}$	$c_{10,2}$	$c_{11,2}$	$c_{12,2}$	$c_{13,2}$	$c_{14,3}$
					Activity or disposal				
n g_1	Capital transfer$_1$	Family living	C_2	O_2	M_2	$Hogs_2$	Grain buying$_2$	Capital transfer$_2$	C_3
	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}
1	$a_{1,6,1}$	$a_{1,7,1}$							
1									
		$a_{6,7,1}$							
2	$-a_{7,6,2}$	$a_{7,7,2}$	$a_{7,8,2}$	$a_{7,9,2}$	$a_{7,10,2}$	$a_{7,11,2}$	$a_{7,12,2}$	$a_{7,13,2}$	
			$a_{8,8,2}$						
				$a_{9,9,2}$					
					$a_{10,10,2}$				
			$a_{11,8,2}$	$a_{11,9,2}$	$a_{11,10,2}$	$a_{11,11,2}$			
			$-a_{12,8,2}$	$-a_{12,9,2}$		$a_{12,11,2}$	$-a_{12,12,2}$		
					$-a_{13,10,2}$	$a_{13,11,2}$			
		$a_{14,7,3}$	$-a_{14,8,3}$	$-a_{14,9,3}$	$-a_{14,10,3}$	$-a_{14,11,3}$	$-a_{14,12,3}$	$-a_{14,13,3}$	$a_{14,14,3}$
			$-a_{15,8,3}$						$a_{15,14,3}$
				$-a_{16,9,3}$					
					$-a_{17,10,3}$				
									$a_{18,14,3}$
									$-a_{19,14,3}$ -
					$-a_{20,10,3}$	$-a_{20,11,3}$			
....					z_{jk}				....
....					$z_{jk}-c_{jk}$				....

	Year 3						
$c_{13,2}$	$c_{14,3}$	$c_{15,3}$	$c_{16,3}$	$c_{17,3}$	$c_{18,3}$	$c_{19,1} = 0$	....
Capital trans-fer$_2$	C_3	O_3	M_3	Hogs$_3$	Grain buying$_3$	Year 1 capital	
P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	....
						$a_{1,19,1}$	
						Disposal activities or identity matrix	
7,13,2							
4,13,3	$a_{14,14,3}$	$a_{14,15,3}$	$a_{14,16,3}$	$a_{14,17,3}$	$a_{14,18,3}$		
	$a_{15,14,3}$						
		$a_{16,15,3}$					
			$a_{17,16,3}$				
	$a_{18,14,3}$	$a_{18,15,3}$	$a_{18,16,3}$	$a_{18,17,3}$			
	$-a_{19,14,3}$	$-a_{19,15,3}$		$a_{19,17,3}$	$-a_{19,18,3}$		
			$-a_{20,16,3}$	$a_{20,17,3}$			
....	....	....	....	....	$z_{18,3} = 0$	$z_{19,1} = 0$	....
....	....	....	....	....	$z_{18,2} - c_{18,2}$	$z_{19,1} - c_{19,1}$	....

year k; z_{jk} the opportunity cost of the j^{th} activity in year k and; $z_{jk}-c_{jk}$ the marginal revenue of the j^{th} activity in year k.

The original input-output matrix for this example is shown in Table 1. In this table, to simplify the presentation, the "disposal" activities (which form an identity matrix) follow the "real" activities. The reversal of identity matrix from the usual configuration used in linear programming does not affect the solution.

2. Restrictions and activities in year 1

The resource restrictions in year 1 are capital, land, June-October labor, feed grain and forage feed (Table 1). The family living restriction (P_{24}) applies to all three years. For convenience, it is included in the resource restrictions of year 1. The resource supplies are denoted by $b_{1,1}$, $b_{2,1}$, $b_{3,1}$, $b_{4,1}$ and $b_{5,1}$. The supply of feed grain and forage feed ($b_{4,1}$ and $b_{5,1}$) both equal zero because in the original matrix, no feed grain or forage feed has been produced. In the P_0 column, b_6 is the amount of capital that may be used for family living in each of the three years. There is no k (year) subscript for b_6 because this value applies to all three years.

Crop activities in year 1 include first year corn, oats and hay which are represented by C_1, O_1 and M_1 respectively. Each crop activity uses capital, land and June-October labor.

Hence, the crop resource requirements (a_{ijk}'s) for capital, land and June-October labor each have a positive sign. For example, the production of one acre of C_1 requires $a_{1,1,1}$ dollars of capital, $a_{2,1,1} = 1.0$ acre of land, and $a_{3,1,1}$ hours of June-October labor. The capital, land and labor coefficients of O_1 and M_1 are based on the same logic as those of C_1. The production of C_1 and/or O_1 adds to the supply of feed grain in year 1. Because feed grain supply is increased rather than decreased, the feed grain coefficients, $a_{4,1,1}$ and $a_{4,2,1}$ (the yield per acre of C_1 and O_1 respectively) are both negative. Likewise, the production of M_1 increases the forage feed supply of year 1. Hence, the forage feed coefficient of M_1 ($a_{5,3,1}$) has a negative sign. Therefore, when production of an activity subtracts from available resource supplies, the a_{ijk} coefficient has a positive sign. But, when production of an activity adds to available resource supplies, the a_{ijk} coefficient has a negative sign.

Hog production (activity P_4) is the only livestock activity in year 1. Activity P_4 requires $a_{1,4,1}$ dollars of capital, $a_{3,4,1}$ hours of June-October labor, $a_{4,4,1}$ bushels of feed grain and $a_{5,4,1}$ tons of hay.

The other activities in year 1 are grain buying (activity P_5), capital transfer (activity P_6) and family living (activity P_7). The purchase of 1 bushel of corn (activity P_5) requires $a_{1,5,1}$ dollars of capital and adds 1 bushel of corn

($-a_{4,5,1}$) to the feed grain supply. To transfer \$1.00 of surplus capital from year 1 to year 2, activity P_6 requires $a_{1,6,1}$ = \$1.00 of capital in year 1. Likewise, the capital coefficient (a_{ijk}) for 1 unit of family living in year 1 is $a_{1,7,1}$ = \$1.00. And, obviously, the family living coefficient for household consumption, $a_{6,7,1}$, must equal 1.0. Since b_6 defines the amount of capital that can be used for family living in year 1, then $\frac{b_6}{a_{6,7,1}} = b_6$. Hence, b_6 = $\left(b_6 \times (a_{1,7,1} = \$1.00)\right)$ dollars of capital are used for family living in year 1.

The net revenues (c_{jk}'s) of the activities in year 1 are shown in the c_{jk} row. The net revenue of an activity equals per unit yield of activity times per unit price of activity minus capital coefficient. For example, the net revenue of C_1 is: yield of C_1 ($a_{4,1,1}$) x price of corn - capital coefficient ($a_{1,1,1}$) = net revenue of C_1 ($c_{1,1}$). Hence, if the cost per acre of C_1 is greater than the total revenue from production C_1, the net revenue will be negative and thus a net cost. Normally, the net revenue for grain buying ($c_{5,1}$) will be a net cost. It is the difference between the purchasing and selling price of corn. The net revenue for capital transfer ($c_{6,1}$) is zero because no cost is assumed in transferring surplus capital from year 1 to year 2. The profit obtained from the capital transfer will be included in the profits of year 2. The net revenue of family living (c_7) is equal to $+M$.[6] Family living is, therefore, forced into the plan of each year (due to the $+M$ value) before any crop or livestock activity.

3. Restrictions and activities in year 2

The resource restrictions in year 2 are: capital, C_1 land, O_1 land, M_1 land, June-October labor, feed grain and forage feed. The capital and land restrictions will be discussed in a later section on inter-year restrictions and activities. In year 2, as in year 1, the supply of feed grain and forage feed equals zero.

Crop activities in year 2 include second year corn, oats and hay which are designated in Table 1 as C_2, O_2 and M_2 respectively. Each requires capital and labor. Hence, the capital and labor coefficients (a_{ijk}'s) of the three crops each have a positive sign. Likewise, each crop requires land for production. In year 2, however, because of the cropping restrictions assumed, C_2 may only follow C_1, O_2 follow O_1 and M_2 follow M_1. Thus, in order that C_2 follow C_1, the C_2 land coefficient ($a_{7,8,2}$) must be opposite the C_1 land restriction for year 2. The logic of the land requirement coefficients will be explained later. For the same reason the O_2 and M_2 land coefficients are opposite the O_1 and M_1 land restrictions. The production of C_2, O_2 and M_2 add to

[6]See: Charnes, Cooper and Henderson. An introduction to linear programming. p. 8.

the feed grain and forage feed supplies of year 2. Hence, the a_{ijk} feed grain coefficients of C_2, O_2 and the forage feed coefficient of M_2 have a negative sign.

In year 2, hog production, grain buying and capital transfer are similar to these same activities in year 1. Therefore, the vectors for these activities have the same a_{ijk} elements as in year 1. Again, in year 2, the price (i.e. net revenue) of family living (+M) forces this activity into the plan before other activities. The capital coefficient for family living in year 2 is $a_{7,7,2}$.

Since the problem in dynamic programming is to maximize the present value of incomes of t future years, future returns need to be discounted. With year 1 representing the "present", and with years 2,3,..., t representing the "future", the net return in year k is discounted. The present discounted value of a net return in year k is computed by: $P.V. = \dfrac{c_{jk}}{(1 + r)^k}$, where P.V. equals present discounted value, c_{jk} the net revenue of the j^{th} activity in the k^{th} year, r the interest rate and, k the year. Hence in Table 2, the c_{jk} values $c_{8,2}$, $c_{9,2}$, $c_{10,2}$, $c_{11,2}$ and $c_{12,2}$ are discounted net revenues or net costs. The c_{jk} value for capital transfer in year 2 ($c_{13,2}$) equals zero. Hence, $c_{13,2}$ cannot be discounted.

4. Restrictions and activities in year 3

The resource restrictions in year 3 are: capital, C_2 land, O_2 land, M_2 land, June-October labor, feed grain and forage feed. Again, the capital and land restrictions of year 3 will be discussed in a latter section on inter-year restrictions and activities. In year 3, as in years 1 and 2, feed grain and forage feed supplies equal zero at the outset. Later, these b_{ik} values must become greater than zero in some iteration.

The crop activities in year 3 are: third year corn (C_3) third year oats (O_3) and third year hay (M_3). Again, each crop requires capital, labor and land for its production. To satisfy the cropping limitations described previously, the C_3 land requirement coefficient ($a_{15,14,3}$) is opposite the C_2 land restriction ($b_{15,3}$). Similarly the O_3 land requirement coefficient ($a_{16,15,3}$) is opposite the O_2 land restriction ($b_{16,3}$) and the M_3 land requirement ($a_{17,16,3}$) is opposite the M_2 land restriction ($b_{17,3}$). In the three years, each land requirement coefficient (a_{ijk}) is equal to 1.0. The production of C_3, O_3 and M_3 contributes to the supply of feed grain and forage feed in year 3. Hence, the coefficients $a_{19,14,3}$, $a_{19,15,3}$ and $a_{20,16,3}$ each have a negative sign.

Hogs and grain buying constitute the other productive enterprises in year 3. The vectors for these activities have the same a_{ijk} elements in year 3 as in years 1 and 2. The cost of family living is deducted from the capital supply of year 3 by the family living capital coefficient $a_{14,7,3}$. No capital transfer activity is required in the activities of year 3 because this is the final year of the plan.

All net revenues (c_{jk}'s) in year 3 are discounted because year 3 is assumed to represent a "future" year.

5. Inter-year restrictions and activities

Inter-year resource restrictions and activity coefficients are necessary in order to make capital, land and other productive factors available in years 2, 3,..., t of the program. That is, in year 1 resources are available for production and consumption. But, in years 2, 3,..., t, the amount of capital, land and feed available depends upon the previous year's production (and consumption). For example, in Table ~~2~~ 1, the supply of capital and land in years 2 and 3 is zero. But, in year 1, $b_{1,1}$ dollars of capital and $b_{2,1}$ acres of land are available for production (or consumption). In all three years, the supply of feed grain and forage feed is zero because in the original input-output matrix, no crops have as yet been produced.

In Table 1 the inter-year resource restrictions in year 2 are: year 1 C_1 land, year 1 O_1 land, year 1 M_1 land and capital. The inter-year resource restrictions for year 3 are: year 2 C_2 land, year 2 O_2 land, year 2 M_2 land and capital. The capital restriction in year 2 and in year 3 is classified as an inter-year restriction because the amount of capital available (b_{ik}) in years 2 and 3 depends upon the previous year's production and consumption.

Inter-year activity coefficients, negative a_{ijk} values, are intermediate products. These intermediate products, including revenue or capital, are outputs of one year which become inputs of the following year. In Table 1 these intermediate products are the a_{ijk} values of year 1's activities which are opposite year 2's resource restrictions. They also include the a_{ijk} values of year 2's activities which are opposite year 3's resource restrictions. However, the vector for family living, activity P_7, which has been included in the activities of year 1, does not include any intermediate products (negative a_{ijk}'s) because it includes only input coefficients (positive a_{ijk}'s). Outputs of capital, land or feed from years 1 to 2, 2 to 3, etc., constitute intermediate products. All intermediate products (a_{ijk}'s) have a negative sign.

As an example of intermediate products, consider the outputs of crop production in year 1. The production of C_1 (activity P_1) makes capital available for year 2. The amount contributed to the capital supply of year 2 ($b_{7,2}$) is $a_{7,1,2}$ dollars (*i.e.* the total revenue obtained from the production of 1 acre of C_1). The output coefficient $a_{7,1,2}$ has a negative sign because it adds to the capital supply of year 2. That is, $-a_{7,1,2}$ = total revenue from C_1 = yield per acre of C_1 ($-a_{4,1,1}$) x price of corn. Similarly, the inter-year capital coefficients $-a_{7,2,2}$ and $-a_{7,3,2}$ represent the total revenue contributed to the capital supply of year 2 by the production of 1 acre of O_1 and 1 acre of M_1.

Under actual farm conditions, the yield of a specific crop in any one year partly depends upon the preceding crop grown. For example, second and third year corn yield much less than first year corn. In Table 1, differences in soil productivity levels due to previous crop production are represented by the intermediate land products ($-a_{ijk}$'s). For example, the production of 1 acre of C_1 in year 1 makes available 1 acre of C_1 land ($-a_{8,1,2}$) for crop production in year 2. Hence, it is necessary to include year 1 C_1 land in the resource restrictions of year 2. Because the intermediate land product $a_{8,1,2}$ is an output, it has a negative sign. The same logic applies to O_1 and M_1 intermediate land products. Thus, the production of 1 acre of O_1 and, the production of 1 acre of M_1 in year 1 makes available 1 acre of O_1 land ($-a_{9,2,2}$) and 1 acre of M_1 land ($-a_{10,3,2}$) for crop production in year 2.

The following mathematical technique permits the transfer of unused (disposal) forage feed from year 1 to year 2: Enter the forage feed a_{ijk} coefficients of M_1 and hogs$_1$ in year 1 twice—once opposite year 1's forage feed restriction and once opposite year 2's forage feed restriction. That is, in the vector for M_1, the forage feed coefficient or yield of M_1 is entered once as $-a_{5,3,1}$ and once as $-a_{13,3,2}$. Both

$-a_{5,3,1}$ and $-a_{13,3,2}$ are intermediate products. The element $-a_{5,3,1}$ is an output in year 1 used in year 1 and the element $-a_{13,3,2}$ is an output from year 1 used in year 2. In order to maintain the correct accounting procedure, the forage feed coefficient of hogs_1 is also entered twice. That is, in the vector for hogs (activity P_4), the forage feed requirement is entered once as $+a_{5,4,1}$ and once as $+a_{13,4,2}$. Both $a_{5,4,1}$ and $a_{13,4,2}$ are inputs rather than outputs and, therefore, neither is an intermediate product. Their only function is to maintain the correct accounting procedure for the transfer of unused hay.

Two other activities in year 1 contribute to the supply of capital in year 2. The first activity is grain buying, activity P_6. The purchase of 1 bushel of corn in year 1 requires $a_{1,5,1}$ dollars of capital. However, in the accounting procedure assumed in this model, no allowance has been made in the livestock (hogs) vector for purchased feed grain. Hence, to maintain the correct accounting procedure, an intermediate capital coefficient, $-a_{7,6,2}$, must be included in the vector for activity P_5. This coefficient, $-a_{7,6,2}$, is equal to the selling price of corn. Also, in terms of absolute values, $a_{1,6,1} \neq -a_{7,6,2}$. The difference is equal to the net cost, $c_{6,1}$, for activity P_5. The second activity is capital transfer (activity P_6). Activity P_6 requires \$1.00 of capital ($a_{1,6,1}$) in year 1 to transfer \$1.00 of capital

($-a_{7,6,2}$) to year 2. That is, coefficient $-a_{7,6,2}$ adds \$1.00 to the capital supply of year 2. Again, $a_{7,6,2}$ has a negative sign because it adds to the capital supply of year 2.

The same technique and logic apply to the inter-year restrictions and activity coefficients of year 2 (i.e., year 2 C_2 land, $-a_{14,8,3}$, $-a_{16,9,3}$, etc.) as applied to the inter-year restrictions and activity coefficients of year 1. That is, the production of 1 acre of C_2 in year 2 makes available $-a_{14,8,3}$ dollars of capital for production or consumption in year 3, $-a_{15,8,3} = 1.0$ acre of C_2 land for crop production in year 3. The same mathematical technique is used to transfer surplus hay from year 2 to year 3. In year 3, there are no intermediate products, because this is the final year of the program.

In Table 1, all c_{jk} values (net revenues) of the disposal activities equal zero in the original matrix because resources are "in non-use". Likewise, the opportunity costs (z_{jk} row) of the various activities all equal zero because, as yet, no crops or hogs have been produced. Therefore, the profit ($z_{jk} - c_{jk}$ in the P_0 column) is zero. Only one $z_{jk} - c_{jk}$ row is necessary because years 1, 2 and 3 are interrelated.

In computing the solution of the expansion model, computation is continued until all marginal revenues ($z_{jk} - c_{jk}$ values) are positive or zero. As long as there are negative $z_{jk} - c_{jk}$ values, profits can be augmented by increasing the quantity of the activity with the negative $z_{jk}-c_{jk}$ value. The simplex method is used to compute the solution. The solution of the dynamic linear programming example presented in Table 1 produces an optimum plan for the 3-year period, after family living has been considered. Profits for each year must be calculated after the optimum plan is obtained because there is only one z_j-c_j row.

D. Algebraic Interpretation of Dynamic Linear Programming

The technique of dynamic linear programming can be presented more precisely in algebraic form. The equations presented in this section outline the algebraic procedure of dynamic linear programming. The equations shown apply to both the expansion and rotation models. In both models, it is possible to assume non-linear relationships between years. However, within each year only linear relationships can be assumed.

An algebraic interpretation of the dynamic linear programming models used in this study is presented in equations (8) where a_{ijk}, x_{jk}, c_{jk} and b_{ik} are as defined previously.

$$a_{1,1,1}x_{1,1}\cdots + a_{1,j,1}x_{j,1} + a_{1,j,2}x_{j,2} + \cdots + a_{1,n,t}x_{n,t} \leqslant b_{1,1}$$

$$a_{2,1,1}x_{1,1}\cdots + a_{2,j,1}x_{j,1} + a_{2,j,2}x_{j,2} + \cdots + a_{2,n,t}x_{n,t} \leqslant b_{2,1}$$

$$\vdots$$

$$a_{i,1,1}x_{1,1}\cdots + a_{i,j,1}x_{j,1} + a_{i,j,2}x_{j,2} + \cdots + a_{i,n,t}x_{n,t} \leqslant b_{i,1}$$

$$a_{i,1,2}x_{1,2}\cdots + a_{i,j,2}x_{j,2} + a_{i,j,2}x_{j,2} + \cdots + a_{i,n,t}x_{n,t} \leqslant b_{i,2}$$

$$\vdots \qquad (8)$$

$$a_{i,1,2}x_{1,2}\cdots + a_{i,j,2}x_{j,2} + a_{i,j,2}x_{j,2} + \cdots + a_{i,n,t}x_{n,t} \leqslant b_{i,2}$$

$$a_{i,1,3}x_{1,3}\cdots + a_{i,j,3}x_{j,3} + a_{i,j,3}x_{j,3} + \cdots + a_{i,n,t}x_{n,t} \leqslant b_{i,3}$$

$$\vdots$$

$$a_{m,1,t}x_{1,t}\cdots + a_{m,j,t}x_{j,t} + a_{m,j,t}x_{j,t} + \cdots + a_{m,n,t}x_{n,t} \leqslant b_{m,t}$$

where $k = 1,2,\ldots,t$

and

$$x_{1,1} \geqslant 0, \;\ldots,\; x_{jk} \geqslant 0, \;\ldots,\; x_{nt} \geqslant 0 \qquad (9)$$

and $f(X) = Z = c_{1,1}x_{1,1} + c_{2,1}x_{2,1} + \cdots + c_{jk}x_{jk} + \cdots + c_{nt}x_{nt}$

(10)

is a maximum. Equations (9) shows that an activity cannot be carried on at a negative level.

If "slack" or "disposal" activities are added to equations (8) the inequalities are changed to equalities. Disposal activities are added to equations (8) by the same procedure as was used to add disposal activities to equations (2) in the static programming model.

In the expansion model, many of the a_{ijk}'s in equations (8) are zero. See Table 1. The a_{ijk}'s for year 1's activities will be zero for all a_{ijk} coefficients for $k > 1$ when year $k + 1$'s activities and restrictions are being considered. For example, where $k = 1$ in equations (8), $a_{1j2}, \ldots a_{1nt} = 0$ as do all a_{ijk} values for years 2, 3, ...,t opposite year 1's restrictions (b_{i1}). The a_{ijk} values are zero because activities and restrictions of year 2, 3, ..., t are separated from activities and restrictions of year 1.

However, if the activities in year 1 include inter-year intermediate products, some of the a_{ijk} coefficients will not equal zero because they are outputs. These outputs (non-zero a_{ijk} coefficients opposite the resource restrictions of year 2) will have a negative sign (see Table 1).

Because there are many zero a_{ijk} elements in the expansion model, it is possible to partition the A matrix (see matrix algebra of the static model, page 8). The partitioning of the A matrix is based on the year of the activity and restriction. That is, the a_{ijk} elements may be partitioned into sub-matrices within the A matrix on the basis of year (k).

Let A_{kk} where, $k = 1,2,\ldots t$, and where the first k refers to the year of the activities and the second k to the year of the restrictions, represent a sub-matrix (within matrix A) of a_{ijk} elements. When the year of the restrictions does not correspond with the year of the activities, the sub-matrix is designated, $A_{k,k+1}$. Then:

$$A_{11} = \begin{vmatrix} a_{1,1,1} & a_{1,2,1} & \cdots & a_{1,j,1} \\ a_{2,1,1} & a_{2,2,1} & \cdots & a_{2,j,1} \\ \vdots & & & \\ a_{i,1,1} & a_{i,2,1} & \cdots & a_{i,j,1} \end{vmatrix}$$

where A_{11} is a sub-matrix of a_{ijk} coefficients for activities in year 1 opposite year 1 restrictions. And

$$A_{12} = \begin{vmatrix} a_{i,1,2} & a_{i,2,2} & \cdots & a_{i,j,2} \\ \vdots & & & \\ a_{i,1,2} & a_{i,2,2} & \cdots & a_{i,j,2} \end{vmatrix}$$

where A_{12} (i.e. $A_{k,k+1}$) is a sub-matrix of non-zero a_{ijk} coefficients for activities in year 1 opposite year 2 restrictions. Thus, ordinarily A_{12} will only consist of intermediate products and, therefore, all a_{ijk} coefficients in A_{12} will have a negative sign. Also $A_{11} \neq A_{12}$.

Consequently, in the expansion model, the A matrix may be written:

$$A = \begin{vmatrix} A_{11} & & & & \\ A_{12} & A_{22} & & & \\ & A_{23} & & & \\ & & A_{kk} & & \\ & & A_{kk+1} & & \\ & & & & A_{tt} \end{vmatrix}$$

In terms of the above matrix notation for the expansion model, the technique of dynamic linear programming—expansion model may be written as shown below:

$$\text{maximize } f(X) = c'X$$

$$\text{subject to } \quad AX \leq B \qquad\qquad x \geq 0$$

$$\text{where } A = \begin{vmatrix} A_{11} & & & & \\ A_{12} & A_{22} & & & \\ & A_{23} & & & \\ & & A_{kk} & & \\ & & A_{kk+1} & & \\ & & & & A_{tt} \end{vmatrix}$$

and where $x = 1,2,..,n$

$c = 1,2,..,n$

$b = 1,2,..,m$

$k = 1,2,..,t$

where X represents a column vector of x_{jk}'s, c a column vector of c_{jk}'s, B a column vector of b_{ik}'s, and c' denotes the transpose of c.

E. Dynamic Linear Programming - Rotation Model

In the rotation model, crop activities are grouped together to form crop rotations. Each crop rotation constitutes one activity. The only difference between the rotation and the expansion models is that, in the former, cropping activities consist of rotations, while in the latter they consist of individual crops. Hence, excluding crops, all aspects of the expansion model are unaltered in the rotation model. Since the technique and logic of the expansion model has been presented previously, this section will only describe differences between the expansion and the rotation models. Table 2 presents the original input-output matrix of a three-year programming example using the rotation model similar to the example discussed for the expansion model. The same notations are used for activities and restrictions in Table 2 as in Table 1.

In Table 2, net revenues of the crop rotations (c_{10}, c_{11} and c_{12}) do not include a k (year) subscript because the rotation includes all three years. Also, land and family living restrictions (b_1 and b_2) do not include a k subscript because these restrictions apply to the three-year period.

In activity P_{10} (C_1-C_2-C_3), the production of C_1 in year 1 contributes capital to year 2 for the production of C_2. The amount of capital made available equals yield of C_1 times price of corn. However, because crops are added together in the rotation model, the intermediate capital product from C_1 production is added to the capital requirement of C_2 production. Hence, only one a_{ijk} capital coefficient is needed which is $\pm\ a_{8,10,2}$. The capital requirement coefficient of C_2 will be $+a_{8,10,2}$ if the capital requirement of C_2 is <u>greater</u> than the intermediate capital product from C_1 production. It will be $-a_{8,10,2}$ if the capital requirement of C_2 is <u>less</u> than the intermediate capital product from C_1 production. Because crops are added together the net revenue of C_1-C_2-C_3(c_{10}) equals net revenue of C_1 plus discounted net revenue of C_2 plus discounted net revenue of C_3. Also, because crops are added together, only one land restriction is needed for all three crops.

In activity P_{12} (M_1-M_2-M_3) surplus hay is transferred from year 1 to year 2 and from year 2 to year 3 by the same technique as was presented in the expansion model. All other aspects of the rotation model in Table 2 have been described previously for the expansion model.

Table 2. Rotation model of dynamic linear programming. Original input-output mat

Resource used or crops and hogs produced each year	Letter to iden- tify	Supply, remain- der or output P_0	$c_{1,1}$ Hogs$_1$ P_1	$c_{2,2}$ Hogs$_2$ P_2	$c_{3,3}$ Hogs$_3$ P_3
Land	P_{13}	b_1			
Family living	P_{14}	b_2			
Yr. 1. Capital	P_{15}	$b_{3,1}$	$a_{3,1,1}$		
Yr. 1. Feed grain	P_{16}	$b_{4,1}$	$a_{4,1,1}$		
Yr. 1. Forage feed	P_{17}	$b_{5,1}$	$a_{5,1,1}$		
Yr. 1. June-Oct. labor	P_{18}	$b_{6,1}$	$a_{6,1,1}$		
Yr. 1. Hog space	P_{19}	$b_{7,1}$	$a_{7,1,1}$		
Yr. 2. Capital	P_{20}	$b_{8,2}$	$-a_{8,1,2}$	$a_{8,2,2}$	
Yr. 2. Feed grain	P_{21}	$b_{9,2}$		$a_{9,2,2}$	
Yr. 2. Forage feed	P_{22}	$b_{10,2}$	$a_{10,1,2}$	$a_{10,2,2}$	
Yr. 2. June-Oct. labor	P_{23}	$b_{11,2}$		$a_{11,2,2}$	
Yr. 2. Hog space	P_{24}	$b_{12,2}$		$a_{12,2,2}$	
Yr. 3. Capital	P_{25}	$b_{13,3}$		$-a_{13,2,3}$	$a_{13,3,3}$
Yr. 3. Feed grain	P_{26}	$b_{14,3}$			$a_{14,3,3}$
Yr. 3. Forage feed	P_{27}	$b_{15,3}$		$a_{15,2,3}$	$a_{15,3,3}$
Yr. 3. June-Oct. labor	P_{28}	$b_{16,3}$			$a_{16,3,3}$
Yr. 3. Hog space	P_{29}	$b_{17,3}$			$a_{17,3,3}$
Opportunity cost	z_j	0	$z_{1,1} = 0$	$z_{2,2} = 0$	....
Marginal revenue	$z_j - c_j$	0	$z_{1,1} - c_{1,1}$	$z_{2,2} - c_{2,2}$	....

put-output matrix for a 3-year program

		Resource for disposal or crop and hog production				
$c_{3,3}$ Hogs$_3$ P_3	$c_{4,1}$ Grain buying$_1$ P_4	$c_{5,2}$ Grain buying$_2$ P_5	$c_{6,3}$ Grain buying$_3$ P_6	$c_{7,1} = 0$ Capital transfer P_7	$c_{8,2} = 0$ Capital transfer$_2$ P_8	c H l
	$a_{3,4,1}$ $-a_{4,4,1}$			$a_{3,7,1}$		a a
	$-a_{8,4,2}$	$a_{8,5,2}$ $a_{9,5,2}$		$-a_{8,7,2}$	$a_{8,8,2}$	a
13,3,3 14,3,3 15,3,3 16,3,3 17,3,3		$-a_{13,5,3}$	$a_{13,6,3}$ $-a_{14,6,3}$		$-a_{13,8,3}$	a
....	....	....	.... z_j	....	....	....
....	....	....	.... $z_j - c_j$	....	....	....

on						
0	$c_9 = +M$	c_{10}	c_{11}	c_{12}	$c_{13} = 0$	....
	Family living	$C_1-C_2-C_3$	$O_1-O_2-O_3$	$M_1-M_2-M_3$	Land	
2	P_9	P_{10}	P_{11}	P_{12}	P_{13}	....
		$a_{1,10,1}$	$a_{1,11,1}$	$a_{1,12,1}$	$a_{1,13}$	
	$a_{2,9}$					
	$a_{3,9,1}$	$a_{3,10,1}$	$a_{3,11,1}$	$a_{3,12,1}$		
		$-a_{4,10,1}$	$-a_{4,11,1}$			
				$-a_{5,12,1}$		
		$a_{6,10,1}$	$a_{6,11,1}$	$a_{6,12,1}$	Disposal	
					activities	
	$a_{8,9,2}$	$\pm a_{8,10,2}$	$\pm a_{8,11,2}$	$\pm a_{8,12,2}$		
		$-a_{9,10,2}$	$-a_{9,11,2}$		or	
				$-a_{10,12,2}$	identity	
		$a_{11,10,2}$	$a_{11,11,2}$	$a_{11,12,2}$		
					matrix	
	$a_{13,9,3}$	$\pm a_{13,10,3}$	$\pm a_{13,11,3}$	$\pm a_{13,12,3}$		
		$-a_{14,10,3}$	$-a_{14,11,3}$			
				$-a_{15,12,3}$		
		$a_{16,10,3}$	$a_{16,11,3}$	$a_{16,12,3}$		
....	....	....	....	$z_{12} = 0$	$z_{13} = 0$	....
....	....	....	....	$z_{12}-c_{12}$	$z_{13}-c_{13} = 0$	.....

IV. FARM PROGRAMMING SITUATIONS

The application of dynamic linear programming in this study is restricted to two owner-operated farms using different levels of conservation. One farm is 160 acres in size; the other, 280 acres. Both farms are located on the Ida-Monona soil association of western Iowa. This study is a continuation of previous studies using these same two farms. For this reason, some of the details regarding the farms and the area are not included in this study.[7]

Optimum five-year plans have been computed for each of the situations which follow:

1. Situation I: 160-acre farm <u>without</u> crop fertilization and <u>without</u> the land being terraced and contoured.
2. Situation II: 160-acre farm <u>with</u> crop fertilization but <u>without</u> the land being terraced and contoured.
3. Situation III: 160-acre farm <u>with</u> crop fertilization and <u>with</u> the land terraced and contoured.
4. Situation IV: 280-acre farm <u>without</u> crop fertilization and <u>without</u> the land being terraced and contoured.

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[7]See Husain, S.M. Aijaz. Optimum resource allocation for erosion control farming on Ida-Monona soils. Unpublished Ph.D. Thesis; Dean, Gerald W., Heady, Earl O., Husain, S.M.A. and E.R. Duncan. Economic optima in soil conservation farming and fertilizer use for farms in the Ida-Monona soil area of western Iowa. Iowa Agr. Exp. Sta. Res. Bul. 455. 1958.

5. Situation V: 280-acre farm with crop fertilization and with the land terraced and contoured.

In each of the above farm situations, average management is assumed for crop and livestock production. Fertilizer is considered to be applied to corn, oats and second-year hay at a single rate.[8] Only one price level is used in programming all situations.

Separate programming matrices were computed for each of the above situations. Each matrix contained activities and restrictions (or resource supplies) for five separate years. The programming solutions for these situations were computed with an IBM Type 650 Magnetic Drum Processing Machine. A modified simplex method developed by Dr. Herman O. Hartley and Mr. Dale D. Grosvenor of the Department of Statistics, Iowa State College, was used.

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[8]Riecken, F.F., Shrader, W.D., Pesek, J.T., Schaller, F.W., Hanway, J.J., Slusher, D.F., Prill, R.C., Ames, Iowa. Information on the single rate of fertilization, (this is the rate necessary to obtain the estimated crop yields used in this study. Both the estimated crop yields and the rate of fertilization were determined by these members of the Department of Agronomy, Iowa State College, Ames, Iowa). 1957. (Private communication).

V. EMPIRICAL METHOD

We now turn to the application of the dynamic linear programming models. The models are employed to obtain optimum five-year plans for a 160-and 280-acre farm in the Ida-Monona soil area of western Iowa. The plans derived for the five situations defined in the previous section represent the most profitable combinations of crops and livestock over a five-year period after considering the annual cost of household consumption. The resource restrictions which limit the optimum plans in each of the five situations studied are presented below.

A. Land

Land is one of the most important resource restrictions in western Iowa. On the 160-acre farm there are 143 acres of cropland and the remaining 17 acres are in farmstead, roads, fences and non-arable land. The 280-acre farm includes 254.5 acres of cropland with the remaining 25.5 acres occupied by farm buildings, roads, fences, gullies and waste land.

Because of the limited number of restrictions accomodated by the IBM 650 "library" program, it was necessary to classify the soils of the two farms into two soil productivity groups; Land A--a low soil productivity soil class; and; Land B--a high productivity soil class. Table 3 shows the classification of cropland by soil type and slope

Table 3. Classification of cropland by soil type and slope of land on the 160 and 280-acre farms[9]

Percent slope interval	Ida	Castana	Monona	Napier	Total
			Acres		
			160-Acre Farm		
0-6	2.8		28.0	33.8	64.6
7-14	6.6		26.1		32.7
15-20	27.4		11.3		38.7
Above 20	1.2		5.8		7.0
Total	38.0		71.2	33.8	143.0
			280-Acre Farm		
0-6	1.2		43.3	53.3	97.8
7-14	7.7		24.7		32.4
15-20	63.0	28.3	7.4		98.7
Above 20	25.6				25.6
Total	97.5	28.3	75.4	53.3	254.5

[9]Source: Husain, S.M. Aijaz. Optimum resource allocation for erosion control farming on Ida-Monona soils. Unpublished Ph.D. Thesis, Ames, Iowa, Iowa State College Library, 1957. (These data have been adjusted to conform with more realistic farm conditions.)

Table 4. Classification of cropland by soil productivity class and convenience of field operations on the 160 and 280-acre farms

Percent slope and land class	Ida	Castana	Monona	Napier	Total
			Soil type		
			percent		
Land A			**160-Acre Farm**		
0-6	10.41				
7-14			11.33		
15-20	53.49		23.74		
Above 20	1.03				
Total	64.93		35.07		100.00
Land B					
0-6			30.48	36.86	
7-14	4.92		22.59		
15-20			5.15		
Above 20					
Total	4.92		58.22	36.86	100.00
Land A			**280-Acre Farm**		
0-6	7.10		4.13	4.68	
7-14			5.09		
15-20	49.79	7.08	5.81		
Above 20	16.32				
Total	73.21	7.08	15.03	4.68	100.00
Land B					
0-6			29.45	36.59	
7-14			14.19		
15-20	0.62	14.98	0.15		
Above 20	4.02				
Total	4.64	14.98	43.79	36.59	100.00

of land on the 160-and 280-acre farms, and Table 4, the composition of Land A and Land B on the two farms by soil type and slope. The 160-acre farm includes 48.6 acres of Land A and 94.4 acres of Land B. On the 280-acre farm there are 124.8 acres of Land A and 129.7 acres of Land B. On the 160 acre farm, Land A consists of 65 percent Ida and 35 percent Monona soils. Land B is made up of 5 percent Ida, 58 percent Monona and 37 percent Napier soils. On the 280 acre farm, Land A consists of 73 percent Ida, 15 percent Monona, 7 percent Castana and 5 percent Napier soils. Land B, of 5 percent Ida, 15 percent Castana, 44 percent Monona and 36 percent Napier.

Because of the difference in soil types and slopes, and, therefore, productivity levels, corn yields are lower on Ida and steep Monona soils than on Napier and Castana soils. In classifying the four soil types into low and high productivity soil groups, most of the Ida and steeper Monona soils were grouped together to form Land A. The majority of the flat Monona plus Napier and/or Castana soils were grouped together to form Land B. However, in grouping the soil types into soil productivity classes, it was necessary to consider field size in order to maintain a reasonable economic size for field operations. Therefore, Land A does not consist entirely of low productivity soils, nor Land B of high productivity soils. Hence, in the Land A soil group, the Castana and Napier soils tend to raise the productivity, whereas, in the Land B soil group, the Ida and steeper Monona soils tend to lower the productivity. It will be noted that land above 20 percent slope is included in both soil groups. This steep land was included in both soil groups because in this area land above 20 percent slope is commonly cropped.

B. Labor and Management

Average management is assumed on both farms for all enterprises. The labor supply on each farm is considered to consist of that provided by the operator and other family members. On the 160-acre farm, the operator supplies 260 hours each month. On the 280-acre farm he supplies 300 hours in November and 260 hours in all other months of the year. The family is considered to supply the following amounts of labor on the 160-acre farm: 26 hours in each of the months, January through April and October through December; 130 hours during each of June, July and August; 40 hours in each of May and September. On the 280-acre farm, family labor accounts for 260 hours during July, August and September, and 52 hours in January through April and October through December. Thus, during the summer months, the 280-acre farm becomes essentially a two-man operation. On both farms, family labor supplies are converted to an operator-equivalent basis. That is, the labor shown is assumed to be, on an hourly basis, as efficient as operator labor. The labor supplies on both farm sizes represent modal labor situations for the two farm sizes.

Total available hours of labor for each month on each farm is presented in Table 5. In this table months have been grouped together to form a sub-group total. This has been done because the limiting labor supply is from March through June and July through November on both farms. The 160-acre farm has 1,262 hours of labor available in March through June, and 1,652 hours available during July through November. On the 280-acre farm for the same periods, there are 1,482 and 2,002 hours available respectively.

C. Capital

One of the most limiting resources encountered by farmers in western Iowa is operating capital. Operating capital may be defined as that capital not invested in machinery, buildings and land. The amount of operating capital available to farmers varies greatly. Even on the same farm, the most profitable combination of crops and livestock differs with the amount of operating capital available for production.

The amount of operating capital available in year 1 on the 160- and 280-acre farm is $9,900 and $14,500 respectively. These capital levels were selected because they allow all land to be cropped in year 1, after a deduction

Table 5. Labor supplied per month on the 160 and 280 acre farms[10]

Month	Total available hours					
	160 acre farm			280 acre farm		
	Operator	Family labor	Total	Operator	Family labor	Total
January	260	26	286	260	52	312
February	260	26	286	260	52	312
March	260	26	286	260	52	312
April	260	26	286	260	52	312
May	260	40	300	260	78	338
June	260	130	390	260	260	520
March-June subtotal			1,262			1,482
July	260	130	390	260	260	520
August	260	130	390	260	260	520
September	260	40	300	260	78	338
October	260	26	286	260	52	312
November	260	26	286	300	52	312
July-November subtotal			1,652			2,002
December	260	26	286	260	52	312

[10] Source: Husain, S. M. Aijaz. Optimum resource allocation for erosion control farming on Ida-Monona soils. Unpublished Ph.D. Thesis, Ames, Iowa, Iowa State College Library. 1957. These data have been adjusted to conform with more realistic farm conditions.

for family living has been made.* That is, in year 1, $9,900 is available for family living, and crop and livestock production on the 160-acre farm and $14,500 is available on the 280-acre farm in each situation studied. In years 2, 3, 4 and 5, the amount of available operating capital depends upon the total revenue from crop and livestock production in the preceding year. Hence, in years 2, 3, 4 and 5, the amount of available operating capital will vary (except by coincidence) on each farm under each situation studied. No capital is borrowed in any of the five years.

D. Farm Buildings

It is assumed that adequate building facilities for crop and livestock production are available on the 160- and 280-acre farms. The 160-acre farm has 720 square feet of hog building space and 1,960 square feet of cattle building space available. On the 280-acre farm 4,600 square feet of hog building space and 1,836 square feet of cattle building space are available. On the 280-acre farm hog space is so plentiful compared to other resources, that it is considered non-limitational. A maximum of 20 litters of hogs and 65 head of cattle can be produced on the 160-acre farm each

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* An annual deduction from operating capital of $3,697 is made for family living in Situations II, III, IV and V. In Situation I, the cost of family living is only deducted from available operating capital in years 2, 3, 4 and 5. The deduction for family living is assumed to represent the cost of family living for a family of two adults and two children.

year under each situation. On the 280-acre farm, a maximum of 61 head of cattle can be produced each year. Adequate facilities for grain and hay storage and for farm machinery are available on both farms.

E. Family Living

In farming, available operating capital is used for both production and household consumption (i.e. operating capital is allocated for family living and farm production from the same fund). Therefore, not all operating capital is available for crop and livestock production. In this study, the annual cost of family living (household consumption) is taken into consideration by deducting $3,697 from available operating capital in each of the five years in Situations II, III, IV and V, and in years 2, 3, 4 and 5 in Situation I. The deduction for family living ($3,697) is assumed to represent the cost of family living for a family of two adults and two children.[11] In year 1 of Situation I, it is assumed that the cost of family living has already been deducted from available operating capital.

In all original programming matrices, family living is both a restriction and a "real" activity. The family living restriction defines (or limits) the amount of operating

[11] Source: Iowa State College Agr. Ext. Service. 1955 family living expenditures of eighty-six Iowa farm families. FM-1231. Ames, Iowa. July, 1956.

capital (i.e. \$3,697) that can be used for household consumption each year. The family living activity, of course, enables family living to come into the optimum five-year plans. In order to make family living come into each plan before any crop or livestock activity, family living has an artificial price (net revenue or c_j value) of \$50 in this study. This artificial price of \$50 means that for every \$1 of operating capital invested in family living, a \$50 profit is returned. The artificial net return performs the same function as the +M (c_j value) in the matrices for the expansion and rotation models presented earlier. That is, operating capital is used for family living before it is used for farm production because family living is forced into the plan before any crop or livestock activities.

F. Machinery and Equipment

It is assumed that on both farms, a complete line of necessary machinery and equipment for crop and livestock production is available.

VI. PRICES

The prices used for net revenues and in computing certain input-output coefficients for each situation studied are given in Table 6. They are the same prices as were used in a previous study[12] and are somewhat higher than those prevailing at the present time.[*] The same price level is assumed to exist in each year of the five years programmed.

A. Fixed Costs

Fixed costs are composed of taxes, insurance, building repairs and depreciation on machinery and buildings and are independent of the level of crop and livestock production. Fixed costs on the 160 and 280-acre farms are estimated at $2,397 and $3,513, respectively.[13] On the 160-acre farm, taxes, interest, insurance and building repairs account for

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[12]Husain. Optimum resource allocation for erosion control farming on Ida-Monona soils., p. 1.

[*]The author realizes that the prices used in this study are somewhat higher than those prevailing at the present time. However, prices fluctuate from day to day and, hence, since the prices used in this study are average adjusted prices, they may reflect future prices as well as any other set of prices that could be used. Also, since these prices have been used in previous studies on these two farms, better comparisons of returns can be made between the various studies. No adjustment has been made in the prices used.

[13]Source: Dean, Gerald W., Heady, Earl O., Husain, S.M.A. and Duncan, E.R. Economic optima in soil conservation farming and fertilizer use in the Ida-Monona soil area of western Iowa. Iowa Agr. Exp. Sta. Res. Bul. 455. 1958.

Table 6. Prices used in determining optimum plans on the 160- and 280-acre farms[14]

Item	Unit	Purchase price ($)	Selling price ($)
Corn (selling)	bu.		1.33
Corn (buying)	bu.	1.43	
Oats	bu.		0.70
Hay	tons	0	0
Alfalfa seed	lb.	0.43	
Brome grass seed	lb.	0.255	
Nitrogen (N)	cwt.	14.40	
Phosphorus (P_2O_5)	cwt.	11.00	
Cattle supplement	cwt.	4.40	
Hog supplement	cwt.	4.40	
Steer feeder calves	cwt.	23.68	
Choice fat cattle	cwt.		26.08
March market hogs	cwt.		18.43
September market hogs	cwt.		19.87
Old sows	cwt.		16.98
Terracing cost	ft.	0.045	
Contouring cost	acre	0.25	

[14]Source: Husain, S. M. Aijaz. Optimum resource allocation for erosion control farming on Ida-Monona soils. Unpublished Ph.D. Thesis, Ames, Iowa, Iowa State College Library. 1957. These data have been adjusted to conform with more realistic farm conditions.

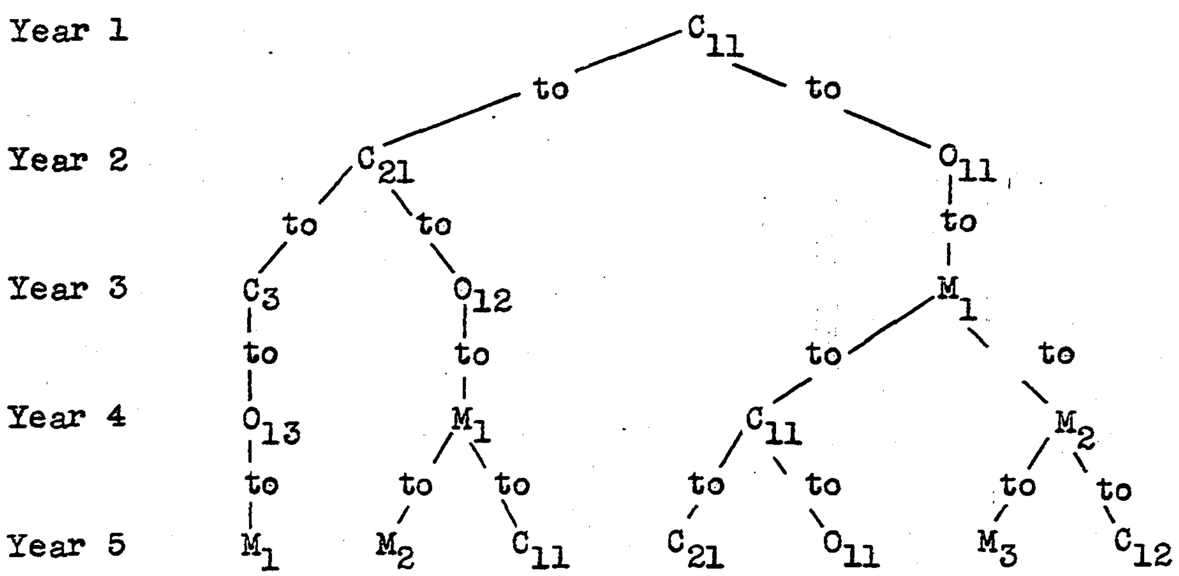
$986 of the total annual fixed costs; machinery depreciation $1,057 and building depreciation $354. Annual fixed costs on the 280-acre farm include $1,500 for taxes, interest, insurance and building repairs; $1,513 for machinery depreciation and $500 for building depreciation.

VII. CROP ENTERPRISES

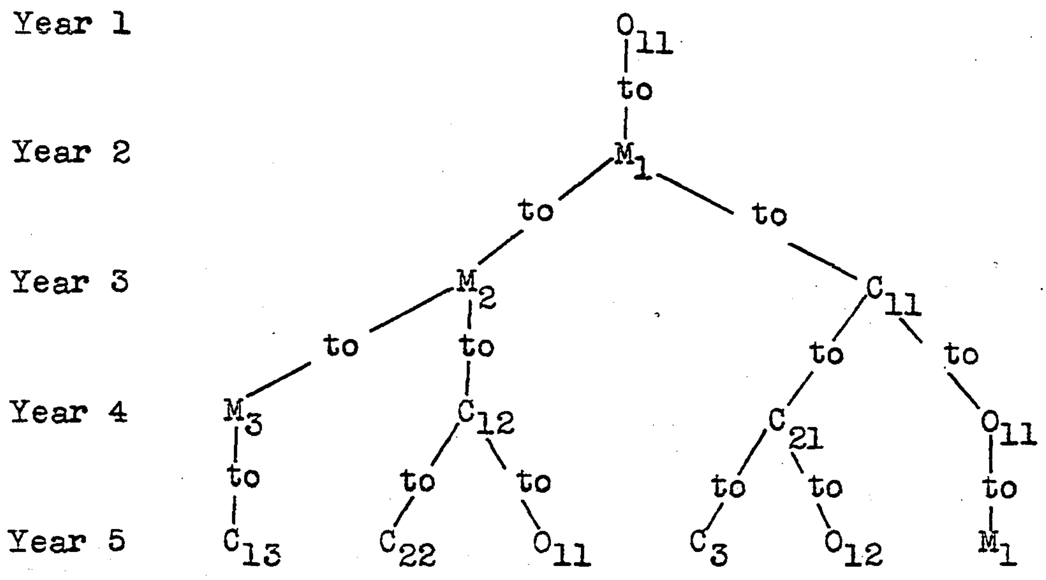
Many farmers do not follow a specific sequence of crops from year to year. Instead, in any particular year, they produce those crops that they think will maximize profits for that year after taking weather conditions, expected prices, preceding crops, yields and feed requirements into consideration. Thus, many farmers produce the crops they think will help maximize profits for a particular year rather than following a predetermined plan.

A. Cropping Possibilities

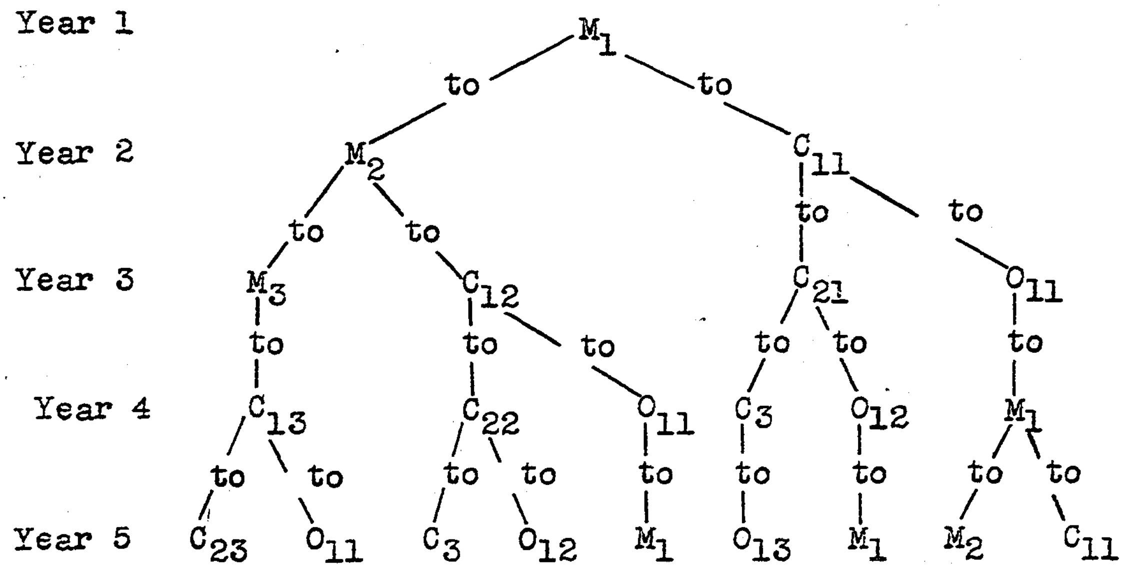
Individual crop enterprises are considered in this study. Within certain limits, all possible combinations of corn, oats and hay over a five-year period are considered. The limits for all situations analyzed are: not more than three consecutive years of corn or hay may be produced; only first year oats may be grown; either corn, oats or hay may be produced in year 1 of any plan and; oats must be used as a nurse crop for hay production in years 2, 3, 4 and 5. It is assumed that by using this approach more realistic five-year plans will be obtained than by only considering several five-year crop rotations as possible cropping enterprises. The number of possible combinations of corn, oats and hay can be explained as follows: Let C_{11} represent first-year corn after one year of hay; C_{12} - first-year corn after two years of hay; C_{13} - first year corn after three years of hay; C_{21} - second -year corn after one year of hay; C_{22} - second-year corn after two years of hay; C_{23} - second-year corn after three years of hay and; C_3 - third-year corn. Also, let O_{11} represent first-year oats after one year of corn; O_{12} - first-year oats after two years of corn and; O_{13} - first-year oats after three years of corn. Denote first-year hay by M_1, and second- and third-year hay by M_2 and M_3 respectively. Starting with first-year corn in year 1, the following 5-year cropping combinations are possible where the symbols show the crops which might be produced each year if each of the crops is included in the optimum plan.



Starting with first-year oats in year 1, the following combinations are possible:



If hay is the crop produced in year 1, the following cropping combinations are possible:



In addition to the cropping combinations shown above, land may be left idle for one or more years. For example, it is possible to have in rotation form: C_{11}-d_2-d_3-d_4-d_5;

C_{11}-C_{21}-d_3-d_4-d_5 etc. where d_k represents idle or disposal land in year k. Hence, there is a total of 52 possible cropping combinations (including d_1-d_2-d_3-d_4-d_5) on either Land A or Land B over the five-year period.

The important aspect of the cropping combinations which emerge from this study is: every possible combination of crops (within the previously defined limits) has been included whether viewed as individual crops in each year of the plan, or as five-year crop rotations. By using this approach, the most profitable crops are grown each year. Furthermore, each crop produces a specific soil productivity level for crop production in the following year. Also, since the same cropping possibilities exist whether viewed as rotations or individual crops, either the expansion or the rotation model will produce the same plan.

All of the above cropping possibilities were included in Situations I, II, III and V. In Situation IV, only d_1-d_2-d_3-d_4-d_5 and five-year rotations having no disposal activities were included in the cropping possibilities. In Situation I, the expansion model was used to obtain the optimum 5-year plan. In Situations II, III, IV and V, the rotation model was used. In Situations I and IV, activities including fertilizer and terracing and contouring were not allowed as cropping possibilities. In Situations II, III and V, all cropping activities were defined to include fertilization and; in Situations III and V terracing and contouring were also allowed.

B. Crop Yields

Table 7 presents the estimated yields of corn, oats and hay by soil type at alternative levels of fertilization and terracing and contouring. The various crops (*i.e.*, C_{11}, C_{12}, etc.) are defined as before. Table 8 presents the yields for the same cropping alternatives on Land A and Land B for the two farms.* The yields are in bushels per acre for corn and oats and in tons per acre for hay. Yields of grain and forage crops are much lower on Land A than on Land B.

Fertilizer application increases crop yields much more than does the use of terracing and contouring. Fertilizer is applied at a single recommended rate. For activities including terracing and contouring, all cropland is considered to be terraced and contoured. Table 9 shows the rates of application of fertilizer for these same crops (*i.e.*, C_{11}, C_{12}, etc.) by soil type, and, Table 10 the cost per acre of fertilizer application for crop production on Land A and Land B for the two farms. Fertilizer costs are much higher on Land A than on Land B for the same crops.

C. Terracing and Contouring

Farmer cost of terracing and contouring is $7.20 per

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* It is assumed that the yield per acre of C_{23} on Land A and Land B is the same as the yield of C_{22}.

Table 7. Estimated corn, oat and hay yields on Ida, Monona, Napier and Castana soils at alternative fertilization in tons per acre

Soil type	Percent slope	Conservation practices / Crop	C_{11}, C_{12}, C_{13} F_0	C_{11}, C_{12}, C_{13} F_1	C_{21} F_0	C_{21} F_1	C_{22} F_0	C_{22} F_1
Ida	7-14	None	15	45	15	42	15	42
		Terracing and contouring	20	52	20	50	20	50
Ida	15-30	None	15	40	15	38	15	38
		Terracing and contouring	20	46	20	44	20	44
Ida	20-	None	12	36	12	34	12	34
		Terracing and contouring	12	37	12	36	12	36
Monona	0-6	None	55	70	48	65	52	65
		Terracing and contouring	60	75	52	70	56	70
Monona	7-14	None	48	60	40	55	44	55
		Terracing and contouring	55	70	46	65	50	65
Monona	15-20	None	44	55	35	50	40	50
		Terracing and contouring	50	64	40	58	45	58
Napier	0-6	None	62	75	54	70	58	70
Castana	15-20	None	50	64	42	58	46	58
		Terracing and contouring	54	68	45	62	50	62

[a]For fertilizer application: F_0 = no fertilizer applied; F_1 - fertilizer applied at a single rate.

ertilization and terracing and contouring levels. Corn and oat yields in bushels per acre, meadow yield

C_{22}	C_3	C_3	O_{11}	O_{11}	O_{12}, O_{13}	O_{12}, O_{13}	M_1, M_2	M_1, M_2	M_3	M_3
			Fertilizer application[a]							
F_1	F_o	F_1	F_o	F_1	F_o	F_1	F_o	F_1	F_o	F_1
42	15	42	15	30	13	30	0.5	2.5	0.5	2.3
50	20	50	18	35	16	35	0.5	2.7	0.5	2.3
38	15	38	14	27	12	27	0.5	2.2	0.5	2.0
44	20	44	16	30	14	30	0.5	2.4	0.5	2.2
34	12	34	12	24	10	24	0.4	1.8	0.4	1.6
36	12	36	12	26	10	26	0.4	2.0	0.4	1.8
65	42	65	33	42	30	42	2.5	3.0	2.2	2.7
70	46	70	35	45	31	45	2.5	3.0	2.2	3.0
55	35	55	30	38	27	38	2.2	2.8	2.0	2.6
65	40	65	32	40	29	40	2.3	2.9	2.0	2.8
50	30	50	28	36	26	36	1.8	2.4	1.8	2.2
58	35	58	30	38	27	38	2.0	2.6	1.8	2.4
70	50	70	35	45	32	45	2.8	3.2	2.8	3.2
58	38	58	32	40	29	40	2.0	2.6	2.0	2.6
62	40	62	32	40	29	40	2.0	2.6	2.0	2.6

Table 8. Corn, oat and hay yields on Land A and Land B by size of farm at alternative fertilization and terracing and co
in tons per acre

Land class	Conservation practices	Crop	C_{11}, C_{12}, C_{13}	C_{11}, C_{12}, C_{13}	C_{21}	C_{21}	C_{22}	C_{22}	C_3
			F_o	F_1	F_o	F_1	F_o	F_1	F_o
Land A	None		25.6	46.3	22.6	43.2	24.2	43.2	20.8
	Terracing and contouring		31.1	53.6	27.7	50.3	29.3	50.3	25.8
Land B	None		53.5	67.6	46.1	62.7	50.0	62.7	41.4
	Terracing and contouring		57.1	72.2	49.2	67.3	53.0	67.3	44.3
Land A	None		24.2	46.2	22.0	43.2	23.2	43.2	20.8
	Terracing and contouring		28.2	51.3	25.9	48.6	27.1	48.6	24.5
Land B	None		53.8	67.9	46.5	62.9	47.3	62.9	41.9
	Terracing and contouring		56.9	71.5	49.0	66.5	53.0	66.5	44.2

[a]For fertilizer application: F_o = no fertilizer applied; F_1 = fertilizer applied at a single rate.

zation and terracing and contouring levels. Corn and oat yields in bushels per acre, meadow yield

C_{22}	C_{22}	C_3	C_3	O_{11}	O_{11}	O_{12}, O_{13}	O_{12}, O_{13}	M_1, M_2	M_1, M_2	M_3	M_3
			Fertilizer application[2]								
F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1
			Yields for 160-acre farm								
24.2	43.2	20.8	43.2	19.2	30.7	17.1	30.7	1.0	2.3	1.0	2.1
29.3	50.3	25.8	50.3	21.3	33.5	19.0	33.5	1.0	2.5	1.0	2.3
50.0	62.7	41.4	62.7	31.9	41.3	29.0	41.3	2.4	3.0	2.3	2.8
53.0	67.3	44.3	67.3	33.2	43.0	30.0	43.0	2.4	3.0	2.3	3.0
			Yields for 280-acre farm								
23.2	43.2	20.8	43.2	18.4	30.2	16.2	30.2	0.9	2.3	0.9	2.1
27.1	48.6	24.5	48.6	19.2	32.7	17.6	32.7	1.0	2.5	0.9	2.3
47.3	62.9	41.9	62.9	32.2	41.4	29.2	41.4	2.4	2.9	2.3	2.8
53.0	66.5	44.2	66.5	33.1	42.7	29.8	42.7	2.4	3.0	2.3	2.9

ngle rate.

Table 9 . Estimated fertilizer requirements of N, P_2O_5 and K for estimated yields (

Crop soil type	C_{11}, C_{12}, C_{13} N P_2O_5 K	C_{21} N P_2O_5 K	C_{22} N P_2O_5 K	(N P
Ida	30 + 80 + 0	60 + 40 + 0	50 + 40 + 0	60 +
Monona	10 + 20 + 0	45 + 20 + 0	35 + 20 + 0	45 +
Napier	10 + 0 + 0	45 + 0 + 0	35 + 0 + 0	45 +
Castana	10 + 20 + 0	45 + 20 + 0	35 + 20 + 0	45 +

yields on Ida, Monona, Napier and Castana soil types

C_3	O_{11}	O_{12}, O_{13}	M_2
N P_2O_5 K	N P_2O_5 K	N P_2O_5 K	N P_2O_5 K
60 + 40 + 0	15 + 40 + 0	20 + 40 + 0	0 + 40 + 0
45 + 20 + 0	10 + 30 + 0	15 + 30 + 0	0
45 + 0 + 0	10 + 0 + 0	15 + 0 + 0	0
45 + 20 + 0	10 + 30 + 0	15 + 30 + 0	0

Table 10. Cost in dollars per acre for fertilizer requirements[a] for estimated crop yields on Land A and Land B by size of farm

Crop	C_{11}, C_{12}, C_{13}	C_{21}	C_{22}	C_3	O_{11}	O_{12}, O_{13}	M_2
Land class			160-acre farm				
Land A	9.80	11.51	10.07	11.51	5.92	6.64	2.96
Land B	3.30	8.08	6.65	8.08	3.61	4.34	0.22
			280-acre farm				
Land A	10.48	11.77	10.33	11.77	5.92	6.38	3.22
Land B	3.27	8.13	6.64	8.13	3.62	4.34	0.20

[a]Fertilizer prices are: N = 14.4¢ per pound; P_2O_5 = 11¢ per pound.

acre for Land A and \$6.01 for Land B. These figures represent only 30 percent of the total costs. The other 70 percent is considered to be paid by the federal government as a soil conservation payment; a common practice in the area. In all situations studied, terracing and contouring costs are charged only against crops produced in year 1 because it is assumed that if terraces are constructed in year 1 they will be present in the other four years; and that farming will be on the contour as required by terracing.

Terracing and contouring costs for Land A and Land B are calculated as follows. It is assumed that Land A has a slope of greater than 8 percent and Land B, a slope of less than 8 percent. One mile of terracing land with a slope of more than 8 percent equals 10 acres protected while one mile of terracing on a slope of less than 8 percent equals 12 acres protected.[15] Terracing costs \$0.045 per foot and contouring \$0.25 per acre. Hence, farmer cost of terracing and contouring one acre of Land A is $\left(\frac{5{,}280 \times \$0.045}{10} + \$0.25\right) .33 =$ \$7.92 and one acre of Land B is $\left(\frac{5{,}280 \times \$0.045}{12} + \$0.25\right) .33$ = \$6.62.

[15]Source: Agricultural conservation program handbook for 1956, Iowa. U.S. Dept. Agr., Agricultural Conservation Program Service, August, 1955.

D. Crop Resource Requirements

Labor and capital coefficients for crops are presented in Table 11. Only seed, fertilizer and terracing and contouring costs are charged against hay production. All other costs incurred in hay production are charged against the livestock enterprise which consumes the hay. For example, the capital requirement for 1 acre of M_1 on Land A or Land B in year 1 when no fertilizer or terracing and contouring are used, is $4.97; the labor requirement is zero because the labor requirement is charged against the livestock enterprise which consumes the hay.

The "fixed" cost per acre for corn, oats and hay represent those cost items (i.e., fuel, seed, insecticides, overhead tractor, fixed machinery, tractor operating and building costs) which are incurred independent of crop yield. These "fixed" costs thus represent costs which vary with the number of acres of crops grown. The variable cost which includes operating costs such as hauling and elevating depicts costs which vary directly with crop yields. For example, the yield of C_{11} on the 160-acre farm on Land B when fertilizer and terracing and contouring are not included is 53.5 bushels per acre. Therefore, the total capital requirement for 1 acre of C_{11} is $17,08 (the fixed cost) plus $0.08 x 53.5 (the variable cost). If C_{11} is fertilized and Land B is terraced and contoured the yield increases to 72.2 bushels per acre and

Table 11. Resource requirements per acre for corn, oats and hay on Land A or Land B on the 160- and 280-acre farms

Item	Unit	Corn[a] (1 acre)	Oats[a] (1 acre)	Hay[b] (1 acre)
Labor requirement	hours			
March	"	----	0.36	----
April	"	1.18	0.90	----
May	"	2.20	----	----
June	"	1.31	----	6.22
Total March-June labor requirement	"	4.69	1.26	6.22
July	"	1.07	1.88	5.30
August	"	----	1.88	----
September	"	0.20	----	4.48
October	"	1.48	----	----
November	"	2.04	----	----
Total July-November labor requirement	"	4.79	3.76	9.78
Fixed cost per acre	$	17.08	13.11	4.79[c]
Variable cost per bushel	$	0.08	0.05	0
Cost of terracing and contouring - Land A	$	7.92	7.92	7.92
Cost of terracing and contouring - Land B	$	6.62	6.62	6.62

[a]Add on a per acre basis: 0.3 hours of April labor for oats, and 0.1 hours of May and June labor for corn when these crops are fertilized.

[b]The labor and variable capital requirements of hay are charged against the livestock enterprise that uses the hay for feed.

[c]Meadow seed cost composed of 8 lbs. of alfalfa seed at 43¢ per pound and 6 lbs. of brome grass seed at 25 1/2¢ per pound.

the total cost to \$17.08 + \$6.62 + \$3.30 (fixed cost) plus \$0.08 x 72.2 (the variable cost). The March-June labor coefficient for 1 acre of C_{11} when fertilizer and terracing and contouring are included is 5.09 hours, and the July-November labor requirement, 4.79 hours.

Since the basic data for crop coefficient computations have been presented in Tables 5, 6, 7, 8, 9, and 10, tables of input-output coefficients and net revenues for each of the cropping systems previously defined have not been computed.

E. Net Revenues of Crops

In all situations studied, the net revenues of individual crops or crop rotations in years 2, 3, 4 and 5 have been discounted because time must be considered in dynamic programming. For example, the net revenue (c_{jk}) of 1 acre of C_{11} on the 160 acre farm on Land B in year 4 when fertilizer and terracing and contouring are not included is:

$$\frac{53.5 \times \$1.33 - (\$17.08 + (\$0.08 \times 53.5))}{(1.0 + 0.06)^4}$$

where the yield of corn is 53.5 bushels per acre, the price, \$1.33 per bushel, and the rate of interest for discounting, 6 percent. Thus, the discounted net revenue of C_{11} is: yield per acre of C_{11} times price of corn minus total cost per acre all divided by the rate of discounting.

VIII. LIVESTOCK ENTERPRISES

The only livestock enterprises included in each situation studied are a 2-litter hog system and deferred-fed calves. As mentioned previously, only these livestock enterprises were considered because previous studies indicated them to be most profitable for the capital levels in this study.

Table 12 presents the basic input-output data and net revenues of the two livestock enterprises.[16] The net revenues of each livestock enterprise are discounted in years 2, 3, 4 and 5 for all situations studied. The resource requirements, however, are the same in all years. For example, the net revenue of a 2-litter hog system in year 4 = $\frac{\$196.48}{(1 + 0.06)^4}$. However, the capital coefficient (cost of production) of a 2-litter hog system in year 4 = \$231.63, the same as in year 1. The cost of forage harvesting is included in the miscellaneous cost item for all situations.

The deferred-fed calf enterprise consists of good choice steer calves purchased in October at 450 pounds, wintered, grazed 60 days on pasture and then full fed to 1,000 pounds and sold in December. In the 2-litter hog system, pigs are farrowed in March and September and are sold six months later

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[16]Husain. Optimum resource allocation for erosion control farming on Ida-Monona soils., p. 1.

Table 12. Basic input-output data and net returns for deferred-fed calves and two litter hog system on a unit basis[a] on the 160- and 280-acre farms

Item	Unit	Deferred-fed calves	Two-litter hog system
Purchase date		October	---
Market date		December	---
Initial weight	lbs.	450	---
Marketing weight	lbs.	1,000	---
Death loss	%	2.5	5.0
Pigs weaned	no.	---	14.160
Pigs sold	no.	---	12.452
Market hogs	lbs.	---	2,739.44
Market sow	lbs.	---	400.00
Total pork	lbs.	---	3,139.44
Feed:			
Corn equivalent	bu.	52	190
Supplement	lbs.	125	1,523
Hay equivalent[b]	tons	2.24	0.70
March-June labor	man-hrs.	7.96	24.13
July-November labor	man-hrs.	17.77	21.42
Building space requirement	sq. ft.	30.0	71.0
Annual cash expense			
Supplement	$	5.50	67.01
Building use	$	2.09	3.25
Power use	$	2.31	20.41
Equipment use	$	2.42	21.03
Miscellaneous cost	$	8.97	26.06
Boar service	$	---	4.00
Death loss	$	2.66	---
Feeder stock	$	106.56	---
Breeding gilt	$	---	62.13

[a]The unit of the deferred-fed calves enterprise is one head. The unit of the two-litter hog system is one sow with two litters of pigs.

[b]Pasture requirements have been converted into tons of hay equivalent.

Table 12. (Continued)

Item	Unit	Deferred-fed calves	Two-litter hog system
Total annual expense	$	130.51	203.89
Investment in equipment	$	13.50	27.74
Total capital outlay	$	144.01	231.63
Net return	$	61.13	196.48

at 220 pounds. The average number of pigs per litter is 7.08. An average death loss after weaning of 5 percent is assumed. One gilt is saved for replacement. The total quantity of pork sold during the year is 3,139 pounds. This includes 400 pounds from the sale of one sow. In Table 11, the pertinent data for computing the per unit requirements used in programming each situation are: bushels of corn equivalent, tons of hay equivalent, March-June and July-November labor, building space requirement, total capital outlay (capital requirement) and net return (c_{jk} value).

IX. ANALYSIS OF RESULTS AND OPTIMUM PLANS

Profit-maximizing, or optimum, 5-year farm plans for the farm situations described earlier are presented in this chapter. Three 5-year plans are shown for the 160-acre farm, and two for the 280-acre farm. In the analysis which follows, the results for each situation studied are discussed separately. Also, a comparison is made of the results within each farm size group. Resource supplies are identical for all situations considered within each farm size group. Thus, variations between plans are due to differences in conservation levels. The same basic cropping and livestock opportunities are available in Situations I, II, III and V. In Situation IV, however, the total number of cropping opportunities is less. As described previously, only 5-year crop rotations and a 5-year land disposal rotation are considered in Situation IV. Hence, in Situation IV, idle or unused land applies to all five years. In other situations it is possible for land to be left idle for five, four, three, two or one year(s).

All optimum five year plans have been computed within the boundaries of available resource supplies. However, corn may be purchased off-the-farm to expand livestock production. In the tables that follow, the "corn surplus or deficit" column for each plan shows the bushels of corn bought or sold each year. A plus sign signifies corn sold, a minus sign indicates corn bought. It is assumed that all hay is produced on the farm. When needed, surplus hay can be transferred from one year to the following year. Similarly, unused capital can be transferred from one year to the next if it can be invested profitably the following year.

In all plans, the net return for each year is given in the "net returns" column. This figure represents the farm's annual return exclusive of family living expenses, (except for year 1 of Situation I). Total net return to the farm is, therefore, net return plus household consumption. In year 1 of Situation I, net return equals total net return because the cost of family living was not deducted from available capital. In all plans, the net return figure in years 2, 3, 4 and 5 is a discounted net return - it is future net return discounted back to its present value.

To obtain net income, fixed costs are subtracted from the net return figure. However, the cost of family living has been deducted from available operating capital in each year of each plan, except year 1 of Situation I. Hence, net income to the farm also includes capital used for household consumption. Net income equals net returns minus fixed costs plus cost of family living. In year 1 of Situation I, net income equals net returns minus fixed costs. Fixed costs (taxes, insurance, building repairs and depreciation on machinery and buildings) on the 160- and 280-acre farm are $2,397 and $3,513 respectively.

Table 13. Situation I: Optimum 5-year plan for a 160-acre farm without fertilizer and withou

Year of plan	Available capital[a]	Dis-posal capital	Optimum combinations of enterprises: Crops: Land class	Crop	Acres	Livestock: Type	Number	T
1	\$ 9,900	\$ 854	A	M_1	48.6	Deferred-fed calves	33 head	
			B	M_1	17.8	2-litter hog system	20 litters	
			B	C_{11}	76.6			
2	\$16,512	\$ 0	A	M_2	48.6	Deferred-fed calves	33 head	Fami
			B	M_2	17.8	2-litter hog system	20 litters	Capi
			B	C_{21}	59.2			
			B	O_{11}	17.4			
3	\$20,079	\$2,856	A	M_3	48.6	Deferred-fed calves	43 head	Famil
			B	M_3	5.9	2-litter hog system	20 litters	
			B	C_{12}	11.9			
			B	O_{12}	59.2			
			B	M_1	17.4			
4	\$18,751	\$1,640	A	C_{13}	48.6	Deferred-fed calves	58 head	Famil
			B	C_{13}	5.9	2-litter hog system	20 litters	
			B	C_{22}	11.9			
			B	M_1	59.2			
			B	C_{11}	17.4			
5	\$19,458	\$5,104	A	C_{23}	48.6	Deferred-fed calves	43 head	Family
			B	C_{23}	5.9	2-litter hog system	20 litters	
			B	C_3	11.9			
			B	C_{11}	23.1			
			B	M_2	36.1			
			B	C_{21}	17.4			

[a]Available capital - capital available for crop and livestock production and family living, e

[b]Net returns before fixed costs are subtracted

tilizer and without terracing and contouring

f enterprises						
c	Other		Corn surplus or deficit (bu.)	Disposal forage (tons)	Limiting resources	Net returns[b]
Number	Type	Value				
33 head 20 litters			+446	0	Land A Land B Forage feed Hog building space	$ 7,466
33 head 20 litters	Family living Capital transfer	$3,697 $3,332	-644	0	Land A Land B Forage feed Hog building space	$ 5,733
43 head 20 litters	Family living	$3,697	-2,662	0	Land A Land B Forage feed Hog building space	$ 4,354
58 head 20 litters	Family living	$3,697	-579	0	Land A Land B Forage feed July-November labor Hog building space	$ 5,324
3 head 0 litters	Family living	$3,697	0	0	Land A Land B Hog building space Forage feed	$ 5,753

d family living, except for year 1.

The data in Tables 13, 14, 15, 16 and 17 have been adjusted to compensate for rounding errors.

In each optimum 5-year plan shown, the same letters and subscripts are used to define crops as described in an earlier section.

A. Situation I: Optimum 5-Year Plan for a 160-Acre Farm without Fertilizer and without Terracing and Contouring

Table 13 presents the optimum 5-year plan for a 160-acre farm that does not use fertilizer or terracing and contouring. Forage crops are grown to provide feed for livestock. The 5-year rotations formed by the expansion model are shown below. These rotations represent the most profitable combinations of crops on Land A and Land B for the 5-year period.

Land class	Rotation	Acres
Land A	M_1-M_2-M_3-C_{13}-C_{23}	48.6
Land B	M_1-M_2-M_3-C_{13}-C_{23}	5.9
	M_1-M_2-C_{12}-C_{22}-C_3	11.9
	C_{11}-C_{21}-O_{12}-M_1-C_{11}	23.1
	C_{11}-C_{21}-O_{12}-M_1-M_2	36.1
	C_{11}-O_{11}-M_1-C_{11}-C_{12}	17.4

In the above crop rotations on Land A and Land B, the same symbols are used to define crop production over the five year period as were used in the section on crop enterprises. For example, in the rotation M_1-M_2-M_3-C_{13}-C_{23} on Land A: first year hay (M_1) is grown in year 1, second year hay (M_2) in year 2, third year hay (M_3) in year 3, first year corn after three years of hay (C_{13}) in year 4 and, second year corn after three years of hay (C_{23}) in year 5. In each of the five years, 48.6 acres are grown.

Over the five-year period, the cheapest source of forage for livestock production is obtained from hay grown on Land A. In other words, hay yields are relatively higher than corn yields on this class of land. On more productive soils such as Land B the reverse is true. Hence, corn is the main crop on Land B. Oats are relatively unprofitable on both classes of land. Their function is to provide a nurse crop for forage. In actuality, many farmers with small acreages would not incorporate more than one rotation. However, as the above plan indicates, Land B is used mostly for growing grain, while Land A is predominantly used for hay. Because of the cropping restriction that not more than three years of continuous corn or hay can be grown, some corn is produced on Land A. Likewise, hay production occurs on Land B. Too, some forage production is required on Land B to provide adequate feed for livestock.

<u>In year 1,</u> no charge is made for family living. Hence, \$9,900 is available for crop and livestock production. Because fertilizer, terracing and contouring are not used, the

capital requirements for crops are lower than in other plans for the 160-acre farm. Consequently, more capital is available for livestock production. Hogs give higher returns to capital than cattle. The maximum hog production allowed by building space is 20 litters. However, the supply of available capital is greater than required for this number of hogs. The next most profitable return from capital is obtained by allocating funds to cattle. The resulting plan for year 1 is 20 litters of hogs and 33 head of deferred-fed calves. Forage feed supplies for livestock are dictated by the cropping program. All of Land A and 17.8 acres of Land B are used for forage production. Remaining acres of Land B are used to grow corn. A total of 446 bushels of corn, not needed for feed, are sold for cash. The limiting resources for this plan are Land A, Land B, forage feed and hog building space. Net return in year 1 is $7,466. If household consumption had been included, less operating capital would have been available and the corresponding net return would be lower than actually shown.

In Table 13, the amount of disposal capital in year 1 is $854. Disposal capital for any one year is the amount of funds not transferred to total supply of available capital (column 2) in the following year. Offhand, in year 2, it may seem unreasonable not to utilize this capital for production. However, the profit function to be maximized in a dynamic programming solution concerns a multi-year period. Crop and livestock production is interrelated through the time span considered. More precisely, in this study, potential land use is directly related to crop production in previous years. For example, third year hay must be followed by first year corn, and third year corn must be followed by first year oats. Consequently, profits are maximized for the five-year period by investing only $16,512 in year 2. If the optimum plan was for years 1 and 2 only, disposal capital in year 1 would have been invested in year 2 because production and corresponding returns in subsequent years would not have been considered.

<u>In year 2</u>, $16,512 of capital is available for family living and crop and livestock production. This figure is obtained by adding year 1 available capital plus net returns minus disposal capital. Since family living is included in the plan for year 2, only $12,815 ($12,815 = $16,512 - $3,697) is actually available for farm production. The same number of deferred-fed calves and hogs are produced in year 2 as in year 1. Also, all of Land A and 17.8 acres of Land B are used to produce the necessary forage feed. However, in year 2, only 59.2 acres of corn are produced. The remainder of Land B is used to grow oats which provide a nurse crop for forage in the following year. The result of substituting oat acres for corn plus decreased yields from second year corn necessitates the purchase of 644 bushels of corn for feed grain. Total capital requirements in year 2 are less

than corresponding available capital; the difference is $3,332 which is transferred to available capital in year 3. This result contrasts the parallel situation in year 1. Unused capital in year 1 could not be profitably invested in year 2; unused capital in year 2 is profitably invested in year 3. Accordingly, capital disposal and capital transfer are specified for these respective years. The limiting resources in the optimum plan for year 2 are: Land A, Land B, forage feed and hog building space. Discounted net return, excluding family living, is $5,733. If family living is included, total net return to the farm for year 2 is $5,733 + $3,697 = $9,430.

In year 3, $20,079 is available for family living and crop and livestock production. The increase in the amount of capital in year 3 and sequence of previous crops permit more cattle production than in years 1 and 2. That is, an increased forage supply combined with increased amount of available capital allows the farm plan to be expanded in year 3. The exact plan is 20 hog litters and 43 head of deferred-fed calves. Correspondingly, crop production is 71.9 acres of hay, 11.9 acres of corn and 59.2 acres of oats. Because of decreased corn acreage and increased number of cattle 2,662 bushels of corn are purchased off the farm. Again, interyear dependence between crops and livestock specifies the optimum plan for any one year. This condition accounts for relatively few acres of corn in year 3. In other words, profits are maximized for the five-year period by purchasing most of the feed grain requirements in year 3, thereby allowing more acres in hay which provide potential corn land in the following years. The limiting resources in year 3 are Land A, Land B, forage feed and hog building space. The discounted net return, excluding family living is $4,354. The decrease in net return in year 3, as compared to year 2, is explained by: fewer acres of corn, and discounting causes the same enterprises in year 3 to have a lower net return than in year 2. However, the major decrease in net return in year 3 is caused by decreased corn acreage.

<u>In year 4</u>, $15,054 ($18,751 - $3,697) is available for crop and livestock production. Again, the amount of available capital in year 4 depends on the amount and types of enterprises produced in year 3. Since the cropping restriction permits only 3 consecutive years of hay production, all of Land A is now used to grow corn. Of the 94.4 acres of Land B, 59.2 acres are used for forage and the remaining acres for corn. Even though less total acres are devoted to hay in year 4 (compared to year 3) higher hay yields on Land B allow 58 head of deferred-fed calves and 20 litters of hogs to be produced. Since the majority of corn is produced on Land A, 579 bushels of corn are purchased for feed. The limiting resources in year 4 are: Land A, Land B, forage feed, hog building space and July-November labor. Labor is now restrictive because of increased cattle production. The discounted

net return, excluding family living, is $5,324. Discounted net return in year 4 is higher than in year 3. This increase results from increased numbers of deferred-fed calves and increased corn acreage. Capital is again non-limitational in year 4.

In year 5, $19,458 is available (from production in year 4) for family living, and crop and livestock production. The only crops grown are corn and hay. However, fewer cattle are produced in year 5 than in year 4, because the cropping sequence in previous years specifies decreased hay acreage in year 5. As usual, the maximum number of hogs are produced. Even with the larger acreage of corn in year 5, as compared to year 4, only enough corn is produced to satisfy the livestock feed grain requirements. The limiting resources are: Land A, Land B, hog building space and forage feed. Unused (disposal) capital in year 5 amounts to $5,094. In all years, where capital is non-limitational (i.e., there is disposal capital), the unused capital may be used to pay for fixed costs. Alternatively, it can be invested off the farm (i.e., in government bonds, etc.), used for expanded consumption or can be regarded as an addition to net returns or net income. Discounted net return is $5,753.

It should be noted that land use in year 4 would allow the same number of forage acres in years 4 and 5. That is, in year 4, 59.2 acres of first year hay are grown. Consequently, 59.2 acres of second year hay could be produced in year 5. But the optimum plan in year 5 specifies 36.1 acres of second year hay with the remaining acres of first year forage land to be allocated to corn production. In other words, profits are maximized over the five-year period by decreasing cattle numbers and hay acreage in year 5 and increasing first year corn acreage.

Over the five-year period, net returns amounted to $28,630. Forage feed is the principal limiting resource in deferred-fed calf production because of low hay yields. The low hay yields are the result of not fertilizing crops. The main restriction for hog production is hog building space. Land is also a limiting resource each year. Capital is not a limiting resource in any of the five years. In year 2, capital transfer would not have occurred had more forage feed been available. Capital is never limitational for two reasons: (1) low capital requirements for crops due to no fertilizer use, (2) limited production of deferred-fed calves due to low forage yields.

B. Situation II: Optimum 5-Year Plan for a 160-Acre Farm with Fertilizer but Without Terracing and Contouring

The main difference between Situation I and Situation II is that in the latter situation, corn, oats and second year hay are fertilized. A second difference is that in Situation

II, family living is included in the optimum plan of <u>each</u> year. All original resource supplies, livestock and crop production possibilities in Situation II are identical with those in Situation I. The optimum five-year plan for Situation II is presented in Table 14.

The following crop rotations represent the most profitable combinations of crops on Land A and Land B for the five-year period.

Land class	Rotation	Acres
Land A	C_{11}-O_{11}-M_1-M_2-M_3	23.3
	M_1-M_2-M_3-C_{13}-C_{23}	10.0
	M_1-M_2-C_{12}-C_{22}-C_3	15.3
Land B	C_{11}-C_{21}-C_3-O_{13}-M_1	40.2
	C_{11}-O_{11}-M_1-C_{11}-C_{21}	35.8
	C_{11}-C_{21}-O_{12}-M_1-C_{11}	18.4

As in Situation I, the optimum five-year cropping plan shows that the high productivity soil (Land B) should be used mainly for corn production, while the low productivity soil (Land A) should be used for forage. That is, hay (and consequently oats, the nurse crop for the hay) should only be produced on Land B to: (supplement forage feed production and/or (2) after three consecutive crops of corn because of the cropping limitation herein assumed.

<u>In year 1</u>, only $6,203 of operating capital is available for crop and livestock production because $3,697 is used for

Table 14. Situation II: Optimum 5-year plan for a 160-acre farm with fertilizer but withou

			Optimum combinations of enterprises					
			Crops			Livestock		
Year of plan	Available capital[a]	Dis-posal capital	Land class	Crop	Acres	Type	Number	
1	$ 9,900	$ 0	A	C_{11}	23.3	Deferred-fed calves	4 head	Fa
			A	M_1	25.3	2-litter hog system	20 litters	
			B	C_{11}	94.4			
2	$15,074	$ 0	A	O_{11}	23.3	Deferred-fed calves	42 head	Fa
			A	M_2	25.3	2-litter hog system	20 litters	
			B	C_{21}	58.6			
			B	O_{11}	35.8			
3	$19,378	$ 0	A	M_1	23.3	Deferred-fed calves	61 head	Fa
			A	M_3	10.0	2-litter hog system	20 litters	
			A	C_{12}	15.3			
			B	C_3	40.2			
			B	M_1	35.8			
			B	O_{12}	18.4			
4	$23,662	$6,647	A	M_2	23.3	Deferred-fed calves	55 head	Fa
			A	C_{13}	10.0	2-litter hog system	20 litters	
			A	C_{22}	15.3			
			B	O_{13}	40.2			
			B	C_{11}	35.8			
			B	M_1	18.4			
5	$21,984	$4,437	A	M_3	23.3	Deferred-fed calves	59 head	Fa
			A	C_{23}	10.0	2-litter hog system	20 litters	
			A	C_3	15.3			
			B	M_1	40.2			
			B	C_{21}	35.8			
			B	C_{11}	18.4			

[a]Available capital = capital available for crop and livestock production - family living.

[b]Net returns before fixed costs are subtracted.

ʻtilizer but without terracing and contouring

of enterprises							
ck		Other		Corn surplus or deficit (bu.)	Disposal forage (tons)	Limiting resources	Net returns[b]
	Number	Type	Value				
s n	4 head 20 litters	Family living	$3,697	+5,319	43.6	Land A Land B Capital Hog building space	$ 8,859
s n	42 head 20 litters	Family living	$3,697	+699	0	Land A Land B Capital Hog building space Forage feed	$ 7,078
; ı	61 head 20 litters	Family living	$3,697	-1,469	41	Land A Land B Capital Hog building space Feed grain	$ 6,768
	55 head 20 litters	Family living	$3,697	-329	0	Land A Land B Hog building space Forage feed July-November labor	$ 6,766
	59 head 20 litters	Family living	$3,697	-430	49.8	Land A Land B Hog building space July-November labor	$ 6,038

ɔn - family living.

household consumption. Again, as in Situation I, hogs give higher returns to capital than cattle. As a result, the maximum number of hog litters allowed by building space is produced. The next most profitable return to capital is obtained by allocating funds to first year corn production. All of Land B plus 23.3 acres of Land A are used to grow corn. Thus, because of higher yields due to crop fertilization, funds are allocated to corn production rather than cattle production. Therefore, capital is first used for family living and corn and hog production, and then the remaining capital is allocated to cattle production. As a result, only 4 head of deferred-fed calves are included in the plan for year 1. As in Situation I, crop production in any one year is interrelated with crop production all other years in the optimum time plan. Thus, 25.3 acres of Land A are used for hay production. More forage feed is produced than is required by the livestock enterprise in year 1. The surplus forage (43.6 tons) is utilized in year 2. Also, a total of 5,319 bushels of corn are not needed for feed and are sold for cash. The limiting resources for this plan are Land A, Land B, capital and hog building space. Capital is the only limiting resource in cattle production, and building space in hog production. Net return in year 1, excluding family living is \$8,859. Crop and livestock production in year 1 provide \$15,062 of available operating capital for year 2. This figure is obtained by adding year 1 available capital to the net return in year 1.

<u>In year 2</u>, since household consumption is included in the plan, only $11,365 ($15,062 - $3,697) is actually available for farm production. The increase in the amount of capital in year 2, plus surplus forage from year 1, permits cattle numbers to be increased to 42 head. As in year 1, hogs are more profitable than cattle, and the maximum number as defined by hog building space is produced. Corresponding crop production is 58.6 acres of corn, 25.3 acres of hay and 59.1 acres of oats. Again, in Situation II, inter-year dependence between crops and livestock specifies the optimum plan for any one year. This condition accounts for the substitution of oat acres for corn acres in year 2. In other words, profits are maximized for the five-year period by growing oats, a nurse crop for forage, thus permitting more forage feed and consequently more livestock to be produced in year 3. Even with increased oat acreage in year 2, corn yields on Land B are high enough to provide <u>more than</u> sufficient feed grain. The surplus corn is sold for cash. The limiting resources are Land A, Land B, capital, hog building space and forage feed. Capital and forage feed limit cattle production and building space, hog production. Discounted net return, excluding family living, is $7,078. The decrease in net return in year 2 is caused by smaller corn acreage plus lower profitability (because of discounting) of the same enterprises in year 2 as compared to year 1.

In year 3, $19,378 is available for family living and crop and livestock production. The increase in the amount of operating capital in year 3 and sequence of previous crops permit more cattle production than in years 1 and 2. The exact plan is 20 hog litters and 61 head of deferred-fed calves. The corresponding crop production is 69.1 acres of forage, 55.5 acres of corn and 18.4 acres of oats. As in previous years, inter-year dependence between crops specifies the optimum plan for any one year. Hence, 41 tons of surplus hay are produced in year 3 that are used for forage feed in year 4. Because of increased livestock and forage acreage in year 3, 1,469 bushels of corn are purchased off the farm for feed grain. In other words, profits are maximized for the five-year period by purchasing feed grain off the farm in year 3, thereby allowing more hay acres which provide potential corn land in years 4 and 5. The limiting resources in the plan for year 3 are: Land A, Land B, capital, hog building space and feed grain. Hog production is limited by hog building space, cattle production by capital and feed grain. Feed grain is a limitational resource in year 3 because operating capital is limitational. Discounted net return, excluding household consumption, is $6,768. If family living is included, total net return to the farm is $6,768 + $3,697 = $10,465.

In year 4, $19,965 ($23,662 - $3,697) is available for crop and livestock production. The amount of available capital in year 4 depends upon the total returns from crop and livestock enterprises in year 3. However, in the plan for year 4, capital is a non-limitational resource. Instead, July-November labor and forage feed restricts cattle production. That is, because of increased grain crop acreage, only 55 head of deferred-fed calves are included in the plan for year 4. As in previous years, 20 hog litters are produced. In other words, hogs and increased grain crop acreage provide higher returns to available July-November labor than does cattle. Consequently, deferred-fed calf numbers are reduced from 61 head in year 3 to 55 in year 4. Even with the increased grain crop acreage in year 4, it is necessary to purchase 329 bushels of corn for feed. Livestock forage feed requirements are satisfied by producing only enough forage in year 4 to supplement surplus forage from year 3. Other limiting resources are Land A, Land B and hog building space. Discounted net return, excluding family living, is $6,766. Net return in year 4 equals net return in year 3 because of increased grain crop acreage in year 4. The surplus capital in year 4, $6,647, may be used to pay for fixed costs. Alternatively, it can be invested off the farm or be used in expanded household consumption or as an addition to net return.

<u>In year 5</u>, $21,984 of operating capital is available (from farm production in year 4) for family living, and crop and livestock production. The only crops grown in year 5 are corn and forage. Oats are not produced in year 5 because

this is the final year of the plan, and therefore, a nurse crop for further forage is not required. More forage is produced than is required for feed by the livestock in year 5. This surplus forage production is a result of the corn cropping limitation previously discussed.

The cropping sequence of the optimum five-year plan specifies 65.3 acres of hay and 79.5 acres of corn in year 5. As usual, the maximum number of hogs is produced. The increased forage acreage (and hence, decreased crop labor requirements) allows more cattle to be produced in year 5 than in year 4. Specifically, the plan calls for 20 litters of hogs and 59 head of deferred-fed calves. Because of the large forage acreage, it is necessary to purchase 430 bushels of corn for feed grain. The limiting resources are Land A, Land B, hog building space and July-November labor. Discounted net return, excluding household consumption, is $6,038. Surplus capital in year 5 amounts to $4,437.

In Situation II, net returns amounted to $35,509 for the five-year period. Hog building space is the only restriction in hog production in each of the five years. Also, hogs are more profitable than cattle, and the maximum number allowed by available hog space is produced each year. Land A and Land B are limitational in each year: all acres of both land classes are fully utilized over the five-year period. Capital is the principal restriction in cattle production in the first three years of the plan, and July-November labor in the last two years. Because of crop fertilization, and therefore higher grain and hay yields, more livestock are included in the optimum five-year plan for this Situation than for Situation I. Also, more corn was sold and less was purchased for feed in Situation II than in Situation I. However, in both situations, Land A was mainly used for hay production, and Land B for corn. Lastly, in Situation II, the optimum five-year plan represents a plan in which the farm firm and the farm household are considered to comprise an interrelated economic unit. Thus, crop and livestock production in each of the five years is interdependent within the farm firm and with household consumption.

C. Situation III: Optimum 5-Year Plan for a 160-Acre Farm with Fertilizer and with Terracing and Contouring

Table 15 presents the optimum 5-year plan for Situation III. In this situation crops are fertilized and the land is terraced and contoured. Situation III differs from Situation II in that terracing and contouring are used. Because of fertilizer use and terracing and contouring, crop yields are higher in Situation III than in the other two situations for the 160-acre farm. As a result of the higher crop yields, fewer acres of Land A or Land B are needed for forage production. Also, since fewer acres are needed for forage production, fewer acres are also needed for oats. The optimum

Table 15. Situation III: Optimum 5-year plan for a 160-acre farm w

Year of plan	Available capital[a]	Dis-posal capital	Crops			Optimum combi
			Land class	Crop	Acres	Type
1	$ 9,900	$ 0	A	M_1	31.0	2-litter hog
			A	C_{11}	17.8	
			B	C_{11}	94.4	
2	$14,258	$ 268	A	M_2	31.0	Deferred-fed
			A	O_{11}	17.8	2-litter hog
			B	C_{21}	59.8	
			B	O_{11}	34.6	
3	$18,311	$ 685	A	C_{12}	31.0	Deferred-fed
			A	M_1	17.8	2-litter hog
			B	O_{12}	26.0	
			B	M_1	34.6	
			B	C_3	33.8	
4	$22,579	$5,630	A	C_{22}	31.0	Deferred-fed
			A	M_2	17.8	2-litter hog
			B	M_1	26.0	
			B	C_{11}	34.6	
			B	O_{13}	33.8	
5	$22,186	$5,192	A	C_3	31.0	Deferred-fed
			A	M_3	17.8	2-litter hog
			B	C_{11}	26.0	
			B	C_{21}	34.6	
			B	M_1	33.8	

[a]Available capital = capital available for crop and livestock pı

[b]Net returns before fixed costs are subtracted.

0-acre farm with fertilizer and with terracing and contouring

Optimum combinations of enterprises				Corn surplus or defici (bu.)
Livestock		Other		
Type	Number	Type	Value	
2-litter hog system	18 litters	Family living	$3,697	+5,972
Deferred-fed calves 2-litter hog system	35 head 20 litters	Family living	$3,697	+1,305
Deferred-fed calves 2-litter hog system	57 head 20 litters	Family living	$3,697	-445
Deferred-fed calves 2-litter hog system	56 head 20 litters	Family living	$3,697	-98
Deferred-fed calves 2-litter hog system	56 head 20 litters	Family living	$3,697	+881

l livestock production - family living.

ontouring

Value	Corn surplus or deficit (bu.)	Disposal forage (tons)	Limiting resources	Net returns[b]
$3,697	+5,972	68.6	Land A Land B Capital	$ 7,913
$3,697	+1,305	0	Land A Land B Hog building space Forage feed	$ 7,101
$3,697	-445	11.6	Land A Land B Hog building space July-November labor	$ 7,239
$3,697	-98	0	Land A Land B Hog building space Forage feed July-November labor	$ 7,309
$3,697	+881	7.5	Land A Land B Hog building space July-November labor	$ 7,577

5-year crop rotations for Situation III are shown below.

Land class	Rotation	Acres
Land A	M_1-M_2-C_{12}-C_{22}-C_3	31.0
	C_{11}-O_{11}-M_1-M_2-M_3	17.8
Land B	C_{11}-C_{21}-O_{12}-M_1-C_{11}	26.0
	C_{11}-O_{11}-M_1-C_{11}-C_{21}	34.6
	C_{11}-C_{21}-C_3-O_{13}-M_1	33.8

As in Situations I and II, the rotations on Land A include more forage crops than rotations on Land B. Forage is grown only to produce forage feed for livestock, and oats as a nurse crop for the forage.

In year 1, \$9,900 is available for family living, and crop and livestock production. Of this, \$3,697 is allocated for household consumption, leaving only \$6,203 for farm production. As in Situations I and II, hogs produce a higher return to capital than cattle. Because of the additional cost of terracing and contouring in year 1, only 18 hog litters are produced in this situation compared to 20 hog litters in year 1 of Situation II. Also capital is so limiting that deferred-fed calves are not included in the plan for year 1. In other words, after family living, capital is first allocated to crop production and the remaining funds used for hog production. The resulting plan in year 1 is 31 acres of hay, 112.2 of corn and 18 litters of hogs. All of

Land B, and 17.8 acres of Land A are used for corn production. The remaining acres of Land A are used for forage. A surplus of 68.6 tons of hay are produced. Also, 5,972 bushels of corn are not required for feed and are sold for cash. Inter-year crop dependence results in surplus hay being produced in year 1 rather than more hogs or cattle. Even though hay production results in a net cost rather than a net revenue in year 1, inter-year crop and livestock production specifies the production of surplus hay to allow increased livestock production in subsequent years. The surplus hay is utilized in year 2. Net return in year 1, excluding family living, is $7,913. Crop and livestock production in year 1 provides $14,116 of available operating capital for year 2. This figure is calculated in the following manner: Available capital in year 2 equals available capital in year 1 plus net return in year 1 minus value of family living in year 1.

In year 2, $10,419 ($14,116 - $3,697 = $10,419) is available for crop and livestock production. The return on capital invested in hog production is much higher than when it is invested in deferred-fed calf production. Hence, capital is first allocated to hog production and then the remaining funds are used for cattle. Because more capital is available in year 2 and surplus forage exists from hay production in year 1, more livestock are produced in year 2. Specifically, hog production is increased to 20 litters and cattle production to 35 head of deferred-fed calves. Corresponding crop production includes 31 acres of hay, 59.8 acres of corn and 52.4 acres of oats. A total of 1,305 bushels of corn are not required for feed and are sold for cash. Again, inter-year crop and livestock interdependence specifies crop production in any one year. Therefore, oat acres are substituted for corn acres to allow increased forage, and consequently increased livestock production in years 3, 4 and 5. The limiting resources in the plan for year 2 are: Land A, Land B, hog building space and forage feed. Hog building space restricts further hog production, and forage feed further cattle production in year 2. It should be noted that capital is a non-limitational resource in year 2. Capital requirements for contouring and terracing are charged only against available operating capital in year 1. Hence, capital was restrictive in year 1. Of course, capital would have been limitational had more deferred-fed calves been included in the plan for year 2. Discounted net return in year 2, excluding household consumption, is $7,101.

In year 3, $18,311 is available (from farm production in year 2) for family living, and crop and livestock production. Increased available capital and larger hay acreage allow the number of cattle to be increased to 57 head. The maximum number of hog litters allowed by building space are produced. Corresponding crop production in year 3 includes 64.8 acres of corn, 26 acres of oats and 52.4 acres of hay. However, more hay is produced than is required by the livestock. The increased hay acreage necessitates the purchase of 445 bushels of corn to meet livestock feed grain requirements. In year 3, capital is again a non-limitational resource. An expanded livestock enterprise causes July-November labor to be restrictive in cattle production. Other limiting resources in year 3 are Land A, Land B, and hog building space. Discounted net return, excluding family living, is $7,239.

In year 4, $22,579 of operating capital is available for family living and farm production. In the optimum 5-year plan for Situation III, crop and livestock enterprises are interdependent between years and within the same year. In year 4, forage acreage is decreased and oat acreage increased. However, because surplus forage is available from year 3, approximately the same number of deferred-fed calves are included in the plan for year 4 as for year 3. As in years 1, 2 and 3, 20 hog litters are produced in year 4. Corresponding crop production includes 65.6 acres of corn, 43.8 acres of forage and 33.8 acres of oats. Oat acres substitute for corn acres to fulfill the cropping restriction previously described and to provide a nurse crop for forage production in year 5. However, not enough feed grain is produced in year 4 to meet the livestock feed requirements, and it is necessary to purchase 98 bushels of corn for feed. Furthermore, only sufficient forage is available in year 4 to produce 20 hog litters and 56 head of deferred-fed calves. The limiting resources are: Land A, Land B, hog building space, forage feed and July-November labor. Labor is limitational in year 4 as in year 3 because of the expanded livestock enterprise. Discounted net return, excluding family living, is $7,309. Surplus capital amounts to $5,630. Unused operating capital may be used to pay for fixed costs, or for expanded household consumption or as an addition to net return.

<u>In year 5</u>, $22,186 is available for family living and farm production. Of this amount, $3,697 is required for household consumption. Exactly the same number of hog litters and deferred-fed calves are included in the optimum plan for year 5 as for year 4. Therefore, the two plans differ only in cropping enterprises. In year 5, only corn and hay are grown because this is the final year of the plan. Because oats are not required for future forage production, corn acreage is expanded. As a result of larger corn acreage, 881 bushels of corn are not used for feed grain and are sold for cash. Again in year 5, crop production is determined by crop enterprises in preceding years. Hence, more forage (7.5 tons) is produced than is required by the livestock enterprises. Under actual farm conditions it may be possible to sell this surplus forage. The limiting resources are: Land A, Land B, hog building space and July-November labor. Discounted net return, excluding family living, is $7,577. Net return in year 5 is higher than in year 4 because of increased corn acreage even though the same enterprises give lower returns due to discounting. Surplus

capital in year 5 amounts to $5,192. Surplus or unused capital in year 5 has the same potential uses as surplus capital in year 4.

Over the five-year period, net returns, excluding family living, amount to $37,139 for Situation III. In this situation, because of fertilizer use and terracing and contouring, yields are higher than in the other two situations for the 160-acre farm. The result is that even though livestock numbers differ between situations, fewer acres of forage are needed to provide forage feed requirements for livestock in Situation III. Because of the additional cost of contouring and terracing in year 1, only crops and hogs are produced. Deferred-fed calves are included in years 2, 3, 4 and 5 because of increased supplies of available operating capital. In Situation III as in Situations I and II, the optimum 5-year plan combines the farm firm and household. The plan thus specifies the best combination of crop and livestock production over a five-year period after the annual cost of family living has been deducted from available operating capital. The farm firm and household are, therefore, considered as one economic unit.

D. Effects of Changes in Conservation Levels on the 160-Acre Farm

Over the five-year period, net returns are lowest when neither fertilizer nor terracing and contouring are included in the plan (Situation I), and highest when they are used (Situation III). However, the increase in net returns is much greater with the addition of fertilizer (Situation II compared to Situation I), than with the addition of terracing and contouring (Situation III compared to Situation II). Also, as fertilizer, and then terracing and contouring are included, total forage acreage over the five-year period decreases. In all plans, returns from forage are relatively greater than from corn on the low productivity soil class (Land A). Conversely, returns are relatively greater from corn production than from forage production on the high productivity soil class (Land B). Hence, a greater proportion of Land A is used for forage than for corn production. The reverse is true for Land B. Crop yields are highest when fertilizer and contouring and terracing are used, and lowest when neither are used. As would be expected, capital is least limitational in year 1 of Situation I, and most limitational in year 1 of Situation III. Finally, when capital is limitational, it appears that net returns to the farm can be increased more by applying fertilizer than by terracing and contouring cropland. If capital is a non-limiting resource, the highest net returns are obtained when fertilizer and contouring and terracing are used.

E. Situation IV: Optimum 5-Year Plan for a 280-Acre Farm without Fertilizer and without Terracing and Contouring

Table 16 presents the optimum 5-year plan for a 280-acre farm without crop fertilization and without terracing and contouring of cropland. This situation is similar to Situation I for the 160-acre farm. In Situation IV idle land applies to all 5 years. Also, in this situation and in Situation V, a hog building space restriction is not included in the resource restrictions of any year (hog space is never a limiting resource).

The optimum 5-year crop rotations for Land A and Land B are given below. In the rotations shown, d_k represents land not cropped in year k (i.e., in disposal).

Land class	Rotation	Acres
Land A	d_1-d_2-d_3-d_4-d_5	113.1
	C_{11}-O_{11}-M_1-M_2-M_3	11.7
Land B	C_{11}-C_{21}-O_{12}-M_1-C_{11}	61.8
	C_{11}-O_{11}-M_1-C_{11}-C_{21}	55.8
	M_1-M_2-C_{12}-C_{22}-C_3	12.1

In this situation, 113.1 acres of Land A are left in disposal for all five years of the plan. Under actual conditions, the disposal land would be put into permanent pasture or rented out. In Situation IV, as in Situations I, II

Table 16. Situation IV: Optimum 5-year plan for a 280-acre farm without fertilizer and w

			Optimum combinations of enterprises				
			Crops			Livestock	
Year of plan	Available capital[a]	Disposal capital	Land class	Crop	Acres	Type	Number
1	$14,500	$ 0	A	d_1	113.1	2-litter hog system	68 litters
			A	C_{11}	11.7		
			B	C_{11}	117.6		
			B	M_1	12.1		
2	$23,477	$1,570	A	d_2	113.1	2-litter hog system	92 litters
			A	O_{11}	11.7		
			B	C_{21}	61.8		
			B	O_{11}	55.8		
			B	M_2	12.1		
3	$32,597	$1,491	A	d_3	113.1	2-litter hog system	112 litters
			A	M_1	11.7		
			B	O_{12}	61.8		
			B	M_1	55.8		
			B	C_{12}	12.1		
4	$37,971	$13,384	A	d_4	113.1	2-litter hog system	96 litters
			A	M_2	11.7		
			B	M_1	61.8		
			B	C_{11}	55.8		
			B	C_{22}	12.1		
5	$32,792	$17,623	A	d_5	113.1	2-litter hog system	72 litters
			A	M_3	11.7		
			B	C_{11}	61.8		
			B	C_{21}	55.8		
			B	C_3	12.1		

[a]Available capital = capital available for crop and livestock production - family livi

[b]Net returns before fixed costs are subtracted.

rtilizer and without terracing and contouring

of enterprises							
ock	Other		Corn surplus or deficit (bu.)	Disposal forage (tons)	Limiting resources	Net returns[b]
Number	Type	Value				
68 litters	Family living	$3,697	0	4.2	Land B Capital Feed grain	$12,674
92 litters	Family living	$3,697	-3,823	0	Land B Forage feed March-June labor	$10,499
112 litters	Family living	$3,697	-9,951	115.2	Land B March-June labor	$ 9,047
96 litters	Family living	$3,697	-5,604	130.8	Land B March-June labor	$ 9,467
72 litters	Family living	$3,697	-436	0	Land B March-June labor	$ 9,794

- family living.

and III, inter-year dependence between crops and livestock specifies the optimum cropping plan in each of the five years. Therefore, profits are maximized over the five-year period by leaving 113.1 acres of Land A in disposal and investing capital and labor resources in other enterprises. In other words, returns to available capital and labor are so low when used on Land A as compared to hog production, that nearly all of Land A is left in disposal. The remainder of Land A, 11.7 acres, is used mainly for forage production over the five-year period. In Situation IV, profits can be maximized over the five-year period by employing Land B mainly for corn production. Forage is only grown to satisfy the cropping restrictions described earlier.

In year 1, \$10,803 (\$10,803 = \$14,500 - \$3,697) of operating capital is available for crop and livestock production. As in previous situations, \$3,697 of capital is used for household consumption. The optimum plan for year 1 includes 68 litters of hogs, 129.3 acres of corn and 12.1 acres of forage. A total of 113.1 acres of Land A are not cropped (i.e., left for disposal). Because hog building space is assumed to be non-limitational, capital invested in hog production returns more than when invested in crop production on 113.1 acres of Land A. Hence, 113.1 acres of Land A are left in disposal and capital is allocated to hogs. Hogs are more profitable than cattle. Therefore, because there is no hog building space limitation, available capital is used for hog production rather than cattle production. In other words, profits are maximized over the five-year period by allocating available operating capital in year 1 to hog production rather than to cattle production and crop production on 113.1 acres of Land A. Cropping restrictions previously described and annual feed requirements of livestock determine the cropping program in any one year and over the five-year period. In year 1, 12.1 acres of Land B are used to grow forage. However, because the forage feed requirements of hogs are very low, more forage is produced than is required for hog production. The surplus forage, 4.2 tons, is utilized in year 2. Since fertilizer and contouring and terracing are not employed in this plan, corn yields are low even on the most productive soil. As a result, all corn produced in year 1 is needed for feed grain for hogs. The limiting resources are Land B, capital and feed grain. Capital is the principal limiting resource. Net return in year 1, excluding family living, is $12,674. Crop and livestock production in year 1 provides $23,477 of available operating capital for year 2.

<u>In year 2</u>, $19,780 is available for farm production (because $3,697 is used for family living). The increased amount of operating capital permits hog production to be expanded to 92 litters. Again, 113.1 acres of Land A are left in disposal because capital and labor resources are more profitably invested in hogs. Furthermore, cattle are not produced in year 2 because hogs are more profitable than

cattle. The cropping plan in year 2 includes 61.8 acres of corn, 67.5 acres of oats and 12.1 acres of hay. Surplus forage feed from year 1 permits the same forage acreage in year 2 as in year 1 even though hog numbers have increased. However, corn acreage is smaller and oat acreage greater in year 2 than in year 1; there has been a substitution of oat acres for corn acres. Oats are grown as a nurse crop for forage production in year 3. Because oats yield less than corn, and corn acreage has decreased, it is necessary to purchase 3,823 bushels of corn for feed grain. Thus, profits are maximized over the five-year period by purchasing corn and leaving the majority of Land A idle in year 2. The limiting resources in year 2 are Land B, forage feed and March-June labor. March-June labor is the principal limiting resource in hog production. Capital is non-limitational in year 2. Surplus capital amounts to $1,570. This surplus capital may be used as previously discussed. Discounted net return, excluding family living, is $10,499. The decrease in net returns is caused by discounting and smaller corn acreage in year 2.

<u>In year 3</u>, $32,597 is available for family living, and crop and livestock production. The increased amount of operating capital plus increased forage acreage allows hog production to be expanded to 112 litters. As in years 1 and 2, cattle are not included in the plan. Corresponding crop production includes only 12.1 acres of corn, 61.8 acres of oats and 67.5 acres of hay. Again, 113.1 acres of Land A

are left in disposal. As a result of the large increase in forage and oat acreage and the large decrease in corn acreage, 9,051 bushels of corn (or almost all the feed grain) must be purchased. In all situations studied, inter-year interdependence between crops and livestock specifies the optimum plan in any one year. Hence, profits are maximized over the five-year period by purchasing almost all the feed grain even though 113.1 acres of Land A could be used to grow corn: It is more profitable to use the limited resources on hog production than on crop production on 113.1 acres of Land A. Because each litter of hogs requires a relatively small amount of hay, a total of 115.2 tons of surplus hay are produced. The limiting resources are Land B and March-June labor. That is, March-June labor is the only restriction in hog production in year 3. Discounted net return, excluding family living, is $9,047. Unused operating capital amounts to $1,491.

In year 4, $37,971 of capital is available. Of this, family living requires $3,697. However, in this year, as in years 2 and 3, capital is not a limiting resource. Increased corn acreage causes the number of hog litters produced to be reduced to 96. Thus, because March-June labor is the critical resource restriction, increased crop labor requirements for corn cause hog production to be decreased. The only other crop grown is hay. In year 4, as in years 1, 2 and 3, 113.1 acres of Land A are left in disposal because labor is more profitably utilized in hog production. However, even with increased corn acreage it is necessary to purchase 5,604 bushels of corn for feed. Forage acreage is also increased in year 4. As a result of increased forage acreage and decreased hog litters, 130.8 tons of surplus hay are produced. Of this amount, only 4.2 tons are used in year 5 to supplement forage production in that year. The limiting resources are Land B and March-June labor. Discounted net return, excluding family living, is $9,467. Surplus capital in year 4 amounts to $13,384.

In year 5, $32,792 is available for family living, and farm production. In year 5, all of Land B is used for corn production. Furthermore, only 11.7 acres of hay are grown on Land A. The remaining acres of Land A are again left in disposal. Due to a large increase in corn acreage in year 5, the number of hogs is reduced to 72 litters because of the March-June labor restriction. However, corn production is not sufficient, and 436 bushels of corn are purchased for feed. Obviously, the large amount of surplus forage feed in year 4 ensures that forage feed is not a limiting resource in year 5. The limiting resources are Land B and March-June labor. Clearly, labor limits hog production. Discounted net return, excluding family living, is $9,794. Surplus capital amounts to $17,623 in year 5.

Over the five-year period, net returns, excluding household consumption, amounted to $51,481 for Situation IV.

Because hogs were more profitable than deferred-fed calves and hog building space was assumed to be non-limitational, only hogs were produced. Furthermore, hog production was more profitable in terms of capital and March-June labor than crop production on 113.1 acres of Land A. Hence, this amount of Land A was left in disposal in each of the five years even though in some of the years almost all the feed grain was purchased off the farm. Under actual farm conditions, many farmers would not leave land idle. The farmer would rent out the disposal land to increase net returns to the farm or put the land into permanent pasture. Finally, surplus capital can be used as discussed earlier. Surplus hay could be sold to increase net returns.

F. Situation V: Optimum 5-Year Plan for a 280-Acre Farm with Fertilizer and with Terracing and Contouring

Table 17 presents the optimum 5-year plan for Situation V -- a 280-acre farm on which crops are fertilized and the cropland is terraced and contoured. In this situation, the same cropping possibilities exist as were present in Situations I, II and III. Hence, it is possible to have idle (or disposal) Land A and Land B in 5, 4, 3, 2 or 1 year(s) of the optimum plan. Also, the same livestock production possibilities exist in Situation V as in the situations discussed previously. Hog building space is assumed to be

Table 17. Situation V: Optimum 5-year plan for a 280-acre farm with fertilizer and with

			Optimum combination of enterpri				
			Crops			Livestock	
Year of plan	Available capital[a]	Dis-posal capital	Land class	Crop	Acres	Type	Number
1	$14,500	$ 0	A	C_{11}	118.4	2-litter hog system	16 litters
			A	M_1	6.4		
			B	C_{11}	129.7		
2	$24,234	$7,187	A	C_{21}	52.8	2-litter hog system	74 litters
			A	d_2	65.6		
			A	M_2	6.4		
			B	O_{11}	91.7		
			B	C_{21}	38.0		
3	$26,219	$3,586	A	C_3	52.8	2-litter hog system	88 litters
			A	d_3	65.6		
			A	C_{12}	6.4		
			B	M_1	91.7		
			B	O_{12}	18.6		
			B	C_3	19.4		
4	$29,676	$13,182	A	d_4	118.4	2-litter hog system	80 litters
			A	C_{22}	6.4		
			B	C_{11}	91.7		
			B	M_1	18.6		
			B	O_{13}	19.4		
5	$27,386	$11,508	A	d_5	124.8	2-litter hog system	78 litters
			B	C_{21}	91.7		
			B	C_{11}	18.6		
			B	d_5	19.4		

[a]Available capital = capital available for crop and livestock production - family liv

[b]Net returns before fixed costs are subtracted.

with terracing and contouring

rprises						
	Other		Corn surplus or deficit	Disposal forage	Limiting	Net
er	Type	Value	(bu.)	(tons)	resources	returns[b]
ters	Family living	$3,697	+13,795	10.1	Land A Land B Capital	$12,976
ters	Family living	$3,697	0	0	Land B Forage feed March-June labor	$10,860
ters	Family living	$3,697	-3,777	243.9	Land B March-June labor	$ 9,289
ers	Family living	$3,697	-328	27.7	Land B March-June labor	$11,467
ers	Family living	$3,697	0	0	Forage feed March-June labor	$10,513

· living.

non-limitational.

The optimum five-year crop rotations on Land A and Land B for Situation V are shown below. Again, d_k represents disposal land in year k.

Land class	Rotation	Acres
Land A	$C_{11}-C_{21}-C_3-d_4-d_5$	52.8
	$C_{11}-d_2-d_3-d_4-d_5$	65.6
	$M_1-M_2-C_{12}-C_{22}-d_5$	6.4
Land B	$C_{11}-O_{11}-M_1-C_{11}-C_{21}$	91.7
	$C_{11}-C_{21}-O_{12}-M_1-C_{11}$	18.6
	$C_{11}-C_{21}-C_3-O_{13}-d_5$	19.4

As in previous situations, these rotations represent the most profitable combinations of crops on Land A and Land B over the five-year period. Thus, crop production in any one year is a function of the cropping restrictions previously described, livestock feed requirements, and crop sequence for the five-year period. In Situation V as in Situation IV, the majority of Land A is left in disposal over the five-year period. This is because capital and labor resources can be more profitably employed in other enterprises than in crop production on Land A. Again, under actual farm conditions the disposal land would be rented out or put in permanent pasture. Unlike other situations studied, corn is the most important crop on the acres of Land A on which crops are grown. The decrease in forage production on the portion of Land A which is cropped is due to the cropping restrictions. This is because at least one year of hay production must occur on Land B over the five-year period. Hence, forage is produced on Land B in the last three years, and on Land A in the first two years of the plan.

In year 1, $15,400 of operating capital is available for for family living, and crop and livestock production. As in previous situations discussed, $3,697 is required for household consumption. In this situation, crops are fertilized and cropland is terraced and contoured. These practices result in higher crop yields for this plan than for the plan for Situation IV. Because of the higher yields, all land is cropped in year 1. The cropping plan includes 6.4 acres of first year hay and 248.1 acres of first year corn. As in the other situations, hogs give a higher return to capital than deferred-fed calves. Hence, available operating capital is allocated to hog production. And, because there is no building space restriction on hog production, only hogs are produced. However, only 16 litters of hogs are produced because of the higher capital requirements of crops (due to fertilization and contouring and terracing). Of the hay produced in year 1, a total of 10.1 tons is not required for hog production. This surplus hay is utilized in year 2. Also, because hog numbers are small in relation to corn acreage, 13,795 bushels of corn are not needed for hog production and are sold for cash. The limiting resources are Land A, Land B and capital. Capital is the only limiting resource in hog production. Net return, excluding family living, is $12,976.

<u>In year 2</u>, $23,477 of operating capital is available from farm production in year 1. Of this amount, $3,697 is required for household consumption. With more operating capital available in year 2, hog production is expanded to 74 litters. As a result of a limited supply of March-June labor and increased hog production, 65.6 acres of Land A are left in disposal (put into permanent pasture or rented out). That is, profits are maximized over the five-year period by allocating available labor to hogs rather than crop production on 65.6 acres of Land A. The remainder of Land A is used for corn and forage production. Because crop yields are higher on Land B, all of Land B is cropped. Specifically, the cropping system in year 2 includes 90.8 acres of corn, 91.7 acres of oats and 6.4 acres of hay. Surplus forage from year 1 makes up the deficit of hay production in year 2. Hence, forage acreage in year 2 is not increased. The oats are grown as a nurse crop for hay in year 3. Because oat acres are substituted for corn acres in year 2, and 65.6 acres of Land A are left idle, only enough grain is produced to meet the feed grain requirements for hogs. The limiting resources in year 2 are Land B, forage feed and March-June labor. Obviously, March-June labor is the principal limiting resource because unused capital and idle land could be used for expanded hog production.

Discounted net return, excluding family living, is $10,860. Net return in year 2 is less than in year 1 because of discounting and smaller corn acreage. Surplus capital in year 2 amounts to $6,630.

In year 3, $26,219 is available for family living, and crop and livestock production. However, this amount of capital is more than sufficient for farm production and household consumption in year 3. Because of increased forage acreage and hence, smaller grain crop labor requirements, hog production is increased to 88 litters. Crop production in year 3 includes 91.7 acres of hay, 78.6 acres of corn and 18.6 acres of oats. Again, 65.6 acres of Land A are left in disposal. The large hay acreage is caused by the cropping restrictions previously discussed. A total of 243.9 tons of surplus hay are produced. Furthermore, the substitution of hay acreage for corn or oat acreage necessitates the purchase of 3,777 bushels of corn for feed grain. As in previous situations discussed, the plan for year 3 may appear illogical, because corn is purchased while land is left idle or used to produce surplus forage. However, under the given resource and cropping restrictions, the plan specifies the best use of resources over the five-year period for maximum profits. In other words, profits would have been lower under any other crop and livestock plan and corresponding allocation of resources. The limiting resources in year 3 are Land B and March-June labor. Again, March-June labor is the only limiting resource in hog production. Discounted net return to the farm, excluding family living, is $9,289.

<u>In year 4</u>, $29,676 of operating capital is available. However, in year 4 as in years 2 and 3, this level of capital is more than sufficient for family living and crop and livestock production. In year 4, corn production is increased to 98.1 acres. As a result of larger March-June crop labor requirements, only 80 hog litters are included in the plan for year 4. However, in spite of reduced hog production, the additional labor requirements for corn necessitate that all of Land A, except 6.4 acres, be left in disposal. The resulting cropping system in year 4 is 18.6 acres of hay, 19.4 acres of oats and 98.1 acres of corn. Even with larger corn acreage, a total of 328 bushels of corn are purchased off the farm for feed grain. The only limiting resources in year 4 are Land B and March-June labor. Discounted net return, excluding family living, is $11,467. Unused operating capital in year 4 amounts to $13,182. Again, this surplus capital may be used to pay for fixed costs, or alternatively, it may be invested off the farm or used for expanded household consumption or as an addition to net return. Similarly, idle Land A may sometimes be rented out for cash to increase net return.

<u>In year 5</u>, $27,386 of operating capital is available for farm production and family living. Because surplus forage feed is available from year 4, corn is the only crop grown in year 5. A total of 110.3 acres of corn are produced on Land

B. The expanded corn acreage, and therefore increased March-June crop labor requirements, results in reduced hog production. In year 5, only 78 litters are produced. Furthermore, the increased March-June labor requirements for corn also necessitate that all of Land A and 19.4 acres of Land B be left in disposal (put into permanent pasture or rented out). It will be noted that 19.4 acres of oats are produced on Land B in year 4. Normally, 19.4 acres of hay would be produced in year 5. However, because the surplus hay from year 4 can be used in year 5, it is not necessary to produce forage in year 5. Hence, the 19.4 acres are left idle in year 5. If corn rather than oats had been produced on the 19.4 acres of Land B in year 4, less March-June labor would have been available for hog production that year. Hence, the plan shown in Table 17 represents the optimum combination of crops and livestock over the five-year period after the cost of family living has been taken into consideration. The limiting resources in year 5 are forage feed and March-June labor. March-June labor is the principal resource limitation in crop and livestock production. Discounted net return in year 5 is $10,513. The decrease in net return in year 5 is due to discounting and reduced hog production. Feed grain is not a limiting resource in year 5 because there is $11,508 of surplus capital available which could be used to buy corn for expanded hog production. Under actual farm conditions, the surplus capital may be used as previously discussed. The

Table 18. Original resource restrictions (supplies) by size of farm for situations I, II, III, IV and V

Item	Unit	Original amount available
160 acre farm		
Land A (low productivity soil)	acre	48.6
Land B (high productivity soil)	acre	94.4
Year 1 capital	$	9,900.00
Hog building space	sq. ft.	720
Cattle building space	sq. ft.	1,960
March-June labor	man-hrs.	1,262
July-November labor	man-hrs.	1,652
Family living	$	3,697.00
280 acre farm		
Land A (low productivity soil)	acre	124.8
Land B (high productivity soil)	acre	129.7
Year 1 capital	$	14,500.00
Hog building space	sq. ft.	4,600
Cattle building space	sq. ft.	1,836
March-June labor	man-hrs.	1,482
July-November labor	man-hrs.	2,002
Family living	$	3,697.00

disposal land would be put into permanent pasture or it may sometimes be rented out for cash to supplement net return.

G. Comparison between Optimum 5-Year Plans for Situations IV and V, and Effects of Changes in Conservation Level on the 280-Acre Farm

Over the five-year period in Situation V, net returns, excluding family living, total $55,105. This represents an increase of $3,624 in total net returns over Situation IV. In Situation V, crop fertilization and terracing and contouring result in higher crop labor requirements. Hence, fewer hogs are produced in Situation V over the five-year period than in Situation IV.

In both five-year farm plans, the majority of Land A is left in disposal (put into permanent pasture or rented out under actual farm conditions) because capital and March-June labor are more profitably employed in hog production than in crop production on Land A. In Situation V, because of crop fertilization and terracing and contouring, higher crop yields permit more corn to be sold and less to be purchased for feed over the five-year period. As a result of higher crop yields, total net returns are highest on the 280-acre farm under Situation V.

In both situations, capital is the principal limiting resource in year 1 and March-June labor in years 2, 3, 4 and 5.

However, capital is most limitational in Situation V because of the additional costs of terracing and contouring and fertilizer. Also, because the 2-litter hog system is more profitable both in terms of capital and labor utilization than deferred-fed calves, only hogs are produced under both situations. The exclusion of cattle production is accounted for by the fact that hog building space is assumed to be non-limitational in both situations. In Situation V, the addition of fertilizer and terracing and contouring result in higher crop yields and hence fewer acres of forage required to produce sufficient forage feed. In both situations, the optimum five-year plan incorporates household consumption and farm production. Therefore, the plans shown represent the best combination of crops and livestock for the farm firm over a five-year period after an allowance has been made for family living.

Since a 280-acre farm employing fertilizer, but not terracing and contouring (i.e., a plan similar to Situation III for the 160-acre farm), was not programmed, it may be stated that: If the same conditions hold regarding fertilizer use and terracing and contouring on the 280-acre farm as on the 160-acre farm, then it is more profitable for the farmer who has limited capital resources to invest his capital in fertilizer rather than terracing and contouring. Also, capital is normally most limiting in the earlier years of each plan. Therefore, if terracing and contouring are adopted, this should only be undertaken in the later years of the plan. In the earlier years, capital should be used to purchase fertilizer.

Finally, three important differences between the optimum five-year plans for Situations IV and V and the optimum five-year plans for Situations I, II and III will be noted. (1) Deferred-fed calves are not included in the optimum plans for Situations IV and V. (2) The majority of Land A is left in disposal (put into permanent pasture or rented out) over the five-year period in Situations IV and V. (3) Part of Land B is left in disposal in year 5 of Situation V. In Situations I, II and III, deferred-fed calves are included in the optimum plans, and crops are grown on all acres of Land A and Land B. The above differences between optimum five-year plans for the two sizes of farms are explained as follows: Hogs are more profitable in terms of capital and labor utilization than cattle. Also, on the 280-acre farm, hogs are more profitable in terms of capital and labor utilization than crop production on the majority of Land A over the five-year period. Likewise, in Situation V, hogs are more profitable in terms of March-June labor than crop production on 19.4 acres of Land B. Hence, because it is assumed that hog building space is a non-limitational resource under Situations IV and V, only hogs are produced. For the same reasons, labor and capital are used for hog production rather than crop production on Land A and Land B. Thus, the majority of Land

A is left in disposal (put into permanent pasture or rented out) over the five-year period, and 19.4 acres of Land B are left in disposal in year 5.

X. INTERPRETATION OF THE FIVE-YEAR PLANS

In this section, the following procedure is used. First, the optimum five-year plans for the 160-acre farm are discussed. Then the corresponding plans for the 280-acre farm are evaluated. The optimum five-year plans are presented in Tables 13, 14, 15, 16 and 17 in the previous chapter.

A. Five-Year Plans for the 160-Acre Farm

The optimum five-year plans for the 160-acre farm are given in Tables 13, 14 and 15. In the plan for Situation I (Table 13), operating capital is a non-limitational resource because: (1) crop fertilization and terracing and contouring are not included, (2) the cost of family living is not deducted from available capital in year 1, and (3) because of low forage yields, cattle production is limited by forage feed rather than by available capital. Hence, annual household consumption has no effect on the cropping or livestock program, because sufficient capital is available each year for household consumption and farm production. Because of low crop yields, the principal limiting resource in cattle production over the five-year period is forage feed. In hog production, it is hog building space. Hogs are more profitable than deferred-fed calves, and the maximum number allowed by building space is produced each year. In the production of crops, the optimum utilization of land in keeping with the cropping limitations is obtained by using the low productivity land for forage production and the high productivity land for corn. However, because crops are not fertilized and terracing and contouring are not used, crop yields are low. For this reason, some of Land B must be used each year to supplement forage production on Land A. Also, because of low yields and use of some of Land B for forage, only 446 bushels of corn are sold over the five-year period, while in year 3, 2,662 bushels are purchased for feed grain. As stated previously, the plan represents the optimum combination of crops and livestock for the five-year period after household consumption is considered. Thus, because the years are interrelated, the cropping system in any one year depends upon livestock feed requirements and crop production over the five-year period. Therefore, by using dynamic linear programming, the plan specified for each year is the one which will result in maximum profits for the five-year period, after household consumption is considered. Because crop yields are low, total net returns over the five-year period are the lowest in this situation of all situations studied.

In the plan for Situation II (Table 14), because crops are fertilized and the cost of household consumption is deducted from available operating capital in year 1, capital becomes limitational. Thus, in the first three years of the plan for this situation, because of the larger capital requirements for crops due to fertilization and the deduction for family living, capital becomes the principal limiting resource in cattle production. In fact, capital shortage is so great in year 1 of Situation II that only 4 head of cattle are produced. However, in years 4 and 5 of the plan for Situation II, capital is not a limiting resource because disposal or unused capital is available due to previous years' production. Instead, because of increased labor requirements due to expanded grain crop and livestock production, July-November labor is the principal limiting resource in cattle production in years 4 and 5. Again, in the plan for this situation, the maximum number of hog litters is produced in each of the five years. Although capital and labor limit cattle production, these resources do not limit crop and hog production, because these latter enterprises are more profitable on farms of the limited resources than cattle. Since the years are interrelated, the cropping system in any one year is a function of livestock feed requirements and crop production over the five-year period. In Situation II as in Situation I, the optimum cropping system (in keeping with the cropping restrictions previously described) is obtained by using Land A mainly for forage production, and Land B for corn production. However, because of fertilization and, therefore, increased yields, less forage acreage is required to meet livestock feed requirements in Situation II than in

Situation I. Net returns over the five-year period are $6,879 greater in Situation II due to higher crop yields than in Situation I. Again, as a result of using dynamic linear programming, the plan specified for each year is the one which results in maximum profits for the five-year period after allowing for family living.

In the plan for Situation III (Table 15), terracing and contouring as well as fertilizer are used. The expense incurred by these procedures (and the cost of family living deducted from operating capital) causes capital in year 1 to become so limitational that no cattle and less than the maximum number of hogs allowed by building space are produced. Again, hogs are more profitable in terms of capital utilization than cattle. All of Land A and Land B are cropped. Thus, household consumption causes a reduction in livestock production in year 1. However, household consumption has no effect on crop production as all available land is cropped. In this study, terracing and contouring must be adopted in year 1 because the cost of terracing and contouring is only deducted from available operating capital in that year. Furthermore, because of the method of programming used in this study, all land must be cropped in year 1 to make possible the crop program specified in years 2, 3, 4 and 5. Thus, terracing and contouring are adopted on all acres of cropland in year 1 because it is profitable due to increased yields over the 5-year period. In years 2, 3, 4 and 5, family living does not affect crop or livestock production because there is surplus of capital available from previous years production. In years 3, 4 and 5, cattle production is limited by July-November labor because of increased crop and livestock labor requirements. Less acres of forage are needed due to increased forage yields. Again, Land A is used mostly for forage production and Land B for crop production. Net returns over the five-year period are higher than in Situation II because of increased yields. However, the increase in net returns from the adoption of terracing and contouring is less than from the adoption of crop fertilization. Again, the technique of dynamic linear programming allows the optimum five-year plan to be determined.

B. Five-Year Plans for the 280-Acre Farm

The optimum five-year plans for the 280-acre farm are given in Tables 16 and 17. In the plan for Situation IV (Table 16), fertilizer and terracing and contouring are not used. Because of this, more capital is available in year 1 for livestock production. Again, in this plan, hogs are more profitable in terms of available capital and labor than cattle, thus only hogs are produced, because hog building space does not restrict hog production. Also, in this plan, hog production is more profitable in terms of capital and labor resources than 113.1 acres of crop products on Land A.

Hence, 113.1 acres of Land A are left in disposal (put into permanent pasture or rented out) and available resource supplies are allocated to hogs rather than crop production. In year 1, because capital is limitational, family living limits hog production. It does not affect crop production, however, even though 113.1 acres are in disposal because there is surplus capital in years 2, 3, 4 and 5 and 113.1 acres of Land A are still left in disposal. That is, the plan in any one year is interdependent with all other years. Hence, Land A would not have been left in disposal in years of surplus capital had family living restricted crop production in year 1. Forage is the most important crop on the Land A which is cropped. That is, Land A provides the cheapest source of forage production. Forage is only grown on Land B to satisfy the cropping restrictions. In years 2, 3, 4 and 5, March-June labor limits crop and livestock production.

In the plan for Situation V (Table 17), capital and therefore household consumption limits hog production in year 1. Again, for the same reasons stated previously, terracing and contouring are adopted on all cropland in year 1. Thus, family living does not affect the adoption of terracing and contouring. Hogs are more profitable than cattle and because there is no hog building space limitation, only hogs are produced each year. Capital is non-limitational in years 2, 3, 4 and 5 because March-June labor limits farm production rather than capital. In fact, in years 2, 3, 4 and 5, an increasing amount of Land A is left in disposal because March-June labor is more profitably employed in hog production than in crop production on Land A. The optimum plan for the five-year period again is one in which the plan for each year is the best possible plan in terms of the five-year optimum. Because of crop fertilization and terracing and contouring crop yields are higher than in Situation IV. As a result, net returns for the five-year period are higher in Situation V than in Situation IV even though fewer hogs are produced. Also in Situation V, because of the cropping limitation, some forage must be grown on Land B. Hence, forage is grown on Land A in years 1 and 2, and on Land B in years 3, 4 and 5. Accordingly, corn is now the most important crop on both Land A and Land B.

The analyses clearly indicates that farm returns can be increased by using fertilizer and terracing and contouring. However, returns are increased more by using fertilizer than by using terracing and contouring. Thus, if the amount of operating capital is very limited, funds should first be allocated to fertilizer. Then the remaining funds should be used for terracing and contouring. If capital is not a limiting resource, net returns can be increased by allocating funds for fertilizer and for terracing and contouring. In all plans shown fertilizer and terracing and contouring were adopted in year 1 because capital was never so limitational that all land could not be cropped.

The plans shown in this study are optimum five-year farm plans after household consumption has been considered. They are not optimum household consumption and farm plans because only a fixed annual charge was made for family living in each plan. That is, a different framework would have had to be used in this study to obtain optimum household consumption and farm plans for a five-year period. Theoretically, it would be possible to derive optimum time plans for farm production and household consumption. To do this, the farm family's preference scale for family living and farm production would need to be assumed. The scale would then dictate whether operating capital would be used for farm production or for family living. Presumably, at very low capital levels, the preference scale would allocate most of the operating capital to a minimum level of family living. As the amount of operating capital increased, farm production and household consumption would be expanded. When operating capital became non-limitational for farm production, it could be used for still further expanded household consumption or it could be invested off the farm in stocks and/or bonds. This too, would be dictated by the preference scale. To be democratic, the preference scale should be composed of weighted preference values of all family members. A series of family living restrictions and activities would be necessary in each original matrix. The preference scale values would be in terms of net revenues or prices. Each c_{jk} value would reflect a certain preference for household consumption for a given capital level. At least one value in each year programmed would have to be high enough to force family living into the optimum time plan.

Also, the plans shown, are intermediate or transition plans rather than final conservation plans. To be final conservation plans, enough years would have had to be programmed that the plan for a series of years was stable. This was not possible because of the limit of 97 restrictions imposed by the IBM 650 "library" program. Also, for the same reason, it was only possible to include one price level and crop fertilization rate and two livestock enterprises in the original input-output matrix of each situation programmed.

Finally, no soil bank payments were included in the original matrices of the situations studied. This would have been possible by having a separate restriction for operating capital available only to forage production and permanent pasture. The limit for this operating capital would be the total amount the government would pay for the adoption of soil bank recommendations. The net revenue, would be the government payment for one acre of forage or permanent pasture.

Further studies need to be made incorporating the above suggestions before optimum time plans for household consumption and farm production (including soil bank payments) can be derived.

XI. SUMMARY

This study was designed to develop a dynamic linear programming model and apply it in determining optimum five-year farm plans but not optimum farm and household consumption plans for alternative conservation situations on the Ida-Monona soil association of western Iowa. Plans were computed for a 160-and 280-acre farm under various conservation alternatives. In all plans, family living (household consumption) is considered before farm production. On both farms, different soil types have been combined to form two land productivity classed: Land A - a low productivity land class and; Land B - a high productivity land class. Adequate farm machinery and hay and grain storage facilities were assumed. The labor supply for competitive enterprises includes that of the operator plus some family labor. Average management is assumed for all crop and livestock enterprises. Only one price level is assumed.

Optimum five-year plans were computed for the following situations; Situation I: 160-acre farm <u>without</u> crop fertilization and <u>without</u> the land being terraced and contoured; Situation II: 160-acre farm <u>with</u> crop fertilization but <u>without</u> the land being terraced and contoured; Situation III: 160-acre farm <u>with</u> crop fertilization and <u>with</u> terracing and contouring; Situation IV: 280-acre farm <u>without</u> crop fertilization and <u>without</u> the land being terraced and contoured;

Situation V: 280-acre farm <u>with</u> crop fertilizer and <u>with</u> terracing and contouring.

On the 160-acre farm, $9,900 of operating capital is available in year 1 of each situation; on the 280-acre farm, $14,500 is available in year 1. In years 2, 3, 4 and 5, the amount of capital available depends upon the total returns from crop and livestock production of the preceding year.

Crop enterprises considered include: all possible combinations and rotations of corn, oats and hay for a five-year period within the limits of; (1) not more than three years production of continuous corn or hay, (2) not more than one year of oats, (3) hay cannot follow corn. Other enterprises included in each situation are: a 2-litter hog system, deferred-fed calves, family living (household consumption), capital transfer and grain buying.

Two dynamic linear programming models are developed in this study. The first model, the expansion model, treats individual crops and non-crop enterprises as activities. The second model, the rotation model, treats crop rotations and single (or individual) non-crop enterprises as activities. Each model permits the simultaneous programming of t years of activities and restrictions. In both dynamic linear programming models, non-linear relationships may be assumed between years but only linear relationships may be assumed within each year.

In the optimum five-year plans for the 160-acre farm, a greater proportion of Land A is used to grow hay than grains. On Land B, hay is grown only to supplement Land A's hay production or, to meet the crop limitation requirements. Fewer acres of Land A and Land B are needed for forage production in Situations II and III than in Situation I because of increased forage yields in the latter situations. Also, in Situations II and III, more corn was sold and less was bought for livestock feed than in Situation I. Under the pricing system used, hogs were more profitable than cattle. For this reason, the maximum number of hogs, allowed by building space or capital restriction were produced each year in Situations I, II and III. Deferred-fed calves were included in each plan on the 160-acre farm to utilize remaining resources after crop and hog production. The number of deferred-fed calves included in the optimum plan for Situation I ranged from 33 to 58 head; for Situation II, from 4 to 61 head and; for Situation III, from 0 to 57 head. Net returns were highest when fertilizer and contouring and terracing were included, and lowest when neither fertilizer nor contouring and terracing were used. Fertilizer use increased net returns more than terracing and contouring.

In the optimum five-year plan for the 280-acre farm hog production was more profitable than crop production on Land A. As a result, nearly all of Land A was left in disposal (put into permanent pasture or rented out) over the five-year period. Also, because it was assumed that hog building space was a non-limitational resource in Situations IV and V, and because hogs were more profitable than cattle, only hogs were produced. The number of hog litters produced each year under Situation IV ranged from 68 to 112; and from 16 to 88 under Situation V. March-June labor was the principle limiting resource in hog and crop production over the five-year period. On the 280-acre farm, net returns were highest when crops were fertilized and the land was terraced and contoured. On both sizes of farms, household consumption did not restrict the adoption of terracing and contouring.

In all optimum five-year farm plans, activities in each of the five years are interrelated. Thus, the cropping system in any one year depends upon the livestock feed requirements and crop production over the five-year period. Therefore, by using the dynamic linear programming models developed in this study, the plan specified for each year in each situation studied is the one which will result in maximum profits for the five-year period, after household consumption has been considered. The plans presented are, therefore, optimum five-year farm plans rather than optimum five-year household consumption and farm plans.

The results of dynamic linear programming provide intermediate farm plans which a farmer can use to move through time from his present plan to an optimum conservation plan. They also provide necessary information for decision making with respect to investment opportunities in conservation and non-conservation practices. They show the crop and livestock systems that will maximize profits both for the present and for the future. The results thus can be used by agriculturalists for making soil conservation recommendations to farmers.

XII. ACKNOWLEDGMENTS

The author is deeply indebted to Dr. Earl O. Heady for suggesting this study and for his valuable guidance and counsel in the preparation of this thesis. The author is also deeply grateful to Dr. Herman O. Hartley for his valuable contribution of the IBM 650 "library" program. The writer also owes a special debt of gratitude to Dr. A. Gordon Ball for his careful reading of the manuscript and for his helpful criticism and advice.

XIII. APPENDICES

A. Appendix A. Original Dynamic Linear Programming Matrices for Situations I, II, III, IV and V

Dynamic linear programming methods for both the expansion and rotation models have been presented in a previous section. The discussion in this section will present the necessary calculations for obtaining the a_{ijk}, b_{ik} and c_{jk} coefficients in the original matrices for the situations studied. The calculations of the coefficients in the expansion model will be discussed first. The coefficients for the crop and livestock enterprises in the various situations have been partly described in previous sections. Situation I employed the expansion model to obtain the optimum 5-year plan. Situations II, III, IV and V used the rotation model.

1. Resource restrictions (b_{ik}'s) for Situations I, II, III, IV and V.

Table 18 presents the original non-zero resource restrictions (b_{ik}'s or P_0 coefficients) for the situation studied. The hog and cattle building space coefficients are entered in years 1, 2, 3, 4 and 5 resource restrictions (P_0 column) of all situations studied. Similarly, March-June and July-November labor restrictions (or supplies) are the same in each of the 5 years. These original resource supplies occur in each year of the original matrix because the same quantity of resource is available each year. Likewise, the same amount of Land A and Land B is available each year.

However, in the P_0 column of the original matrix of the expansion model, the only non-zero values for Land A and Land B occur in year 1. In years 2, 3, 4 and 5 all Land A and Land b resource restrictions are zero (see algebraic model p.14). The land restrictions in years 2, 3, 4 and 5 are zero, because in the original matrix no crops have been produced. Hence, no land has been transferred from year 1 to years 2, 3, 4 and 5 and therefore, all Land A and Land B restrictions must have a zero value in years 2, 3, 4 and 5 in the original matrix of Situation I. In the rotation model, however, land restrictions (for Land A and Land B) need only be entered in the P_0 column once. In this model, crops are in rotation form, and therefore, only one Land A and one Land B resource restriction is necessary for all rotations.

In the expansion model, only individual crops are considered. Hence, each crop provides a different soil productivity for the following year's crop. Therefore, as many land restrictions are necessary as there are different crops on different land classes. For example, in year 2, first year corn on Land A is considered a different crop (produces a different soil productivity level for the following year) than first year corn on Land B. Hence, separate Land A and Land B restrictions are necessary in year 3 for first year corn on Land A, and first year corn on Land B. Likewise, in year 3, there will be individual restrictions for second year corn on Land A and, second year corn on Land B. In other words, these soils are of a different productivity level than the soils from first year corn on Land A and first year corn on Land B.

In the original matrices for both the rotation and expansion models, the capital supply in the P_0 column for years 2, 3, 4 and 5 equals zero. In year 1 of Situations I, II and III, the capital coefficient (b_{11}) has a value of \$9,900. In year 1 of Situations IV and V, it equals \$14,500. The capital restriction coefficients for years 2, 3, 4 and 5 are zero in the original matrices of the various situations because, as yet, no activity has entered into the plan. Thus, since the amount of available capital in year 2 depends on what was produced in year 1, available capital in year 2 equals zero, because, as yet, nothing has been produced in year 1. Similarly, amount of available capital in year 3 depends on crop and livestock production in year 2. Thus, if the amount of capital available in year 2 equals zero, the amount of capital available in year 3 equals zero. No capital is available in the original matrices in years 4 and 5 for the same reason. However, a capital restriction for years 2, 3, 4 and 5 must be entered in the original P_0 column of each dynamic program matrix, even though the value is zero.

The annual limit on family living for the 5-year period is entered once in the P_0 column of the expansion or rotation model. A deduction for family living will occur in each of the 5 years, even though only one restriction for family living is entered in the original P_0 column. This is accomplished through the family living activity in the matrix. The family living vector has capital requirement coefficients (a_{ijk} = 1.0) opposite each year's capital resource restriction. The net revenue (c_{jk}) for the family living activity (vector), has an artificially high value. This artificial value forces family living into the plan before all other activities. Thus, capital is used for family living each year before it is used for any productive enterprise.

2. Original matrix for Situation I

a. Crop enterprises. The cropping possibilities considered in Situation I were outlined previously in the section on cropping opportunities. For example, in year 1, the following individual crops may be produced: C_{11} on Land A, C_{11} on Land B, O_{11} on Land A, O_{11} on Land B, M_1 on Land A, M_1 on Land B; where, as was used previously, C_{11}, O_{11} and M_1 denote first year corn, oats and hay. The capital requirement (a_{ijk}) for 1 acre of C_{11} on Land A is: \$17.08 (constant cost per acre) + \$0.08 (variable cost per acre) x 25.6 (yield per acre) = \$19.13. The net revenue (c_{jk}) for 1 acre of C_{11} on Land A is 25.6 (yield per acre) x \$1.33 (price of corn) - \$19.13 (capital requirement) = \$14.92. March-June and July-November labor requirements (a_{ijk}'s) are 4.69 and 4.79 (see Table 13, p. 66). The coefficient (a_{ijk})

for feed grain is -25.6. The feed grain coefficient has a minus sign because the production of 1 acre of C_{11} adds to the quantity of feed grain available in year 1. That is, the production of 1 acre of C_{11} in year 1 adds 25.6 bushels of corn to the feed grain supply of year 1. The C_{11} Land A requirement in year 1 (a_{ijk} value opposite Land A restriction in year 1) is 1.0. However, the land requirement for C_{11} on Land A in year 2's restrictions (a_{ijk}), is -1.0. This latter land requirement (a_{ijk}), is an intermediate product. In year 1, 1.0 acre of Land A is necessary to produce 1.0 acre of C_{11}. Hence, the land requirement (a_{ijk}) of C_{11} opposite year 1's Land A restriction (b_{i1}), is +1.0. But, the production of 1 acre of C_{11} in year 1 makes 1 acre of C_{11} on Land A available for production of crops in year 2. Hence, the land transfer coefficient (intermediate product) of C_{11} opposite year 2's C_{11} Land A restriction (b_{i2}) is -1.0. The same logic and method applies to all other crops in Situation I. Therefore, all land requirements of crops are +1.0 when the year in which the crop is grown coincides with the year of the land restriction and, all intermediate land products are -1.0 when the year in which the crop is grown does not coincide with the year of the land restriction.

The capital coefficient (a_{ijk}), for C_{11} on Land A in year 1 opposite the year 2's capital restriction (b_{i2}), equals (-25.6 x \$1.33) = -\$34.04. This capital coefficient has a minus sign because the production of one acre of C_{11} on Land A in year 1 adds $34.04 to the supply of capital in year 2. The same logic and method applies to all other intermediate capital products in Situation I. That is, all intermediate capital coefficients have a minus sign, because all add to the following year's capital supply.

The method of transferring unused hay of one year to the following year was described in an earlier section on crops and livestock. In the matrix for Situation I, (and all other situations studied), no labor charge was included for hay production. Therefore, in the original matrices, all labor requirements (a_{ijk}'s), for hay (M_1, M_2, M_3), equal zero. The labor requirements for hay are charged against the livestock enterprise that consumes the hay. The net revenues for hay are all net costs because hay has no sale price. Hence, in year 1, the net revenue for M_1 equals -$4.97 (i.e., $0 (total return from M_1) minus $4.97 (capital coefficient for M_1)).

b. Other enterprises. The following enterprises are considered in each year of all situations studied: 2-litter hog system, deferred-fed calves, grain buying and, capital transfer. In addition to these enterprises, family living is included in the activities of years 1, 2, 3, 4 and 5 for Situations II, III, IV and V and; in years 2, 3, 4 and 5 for Situation I. The enterprises described in this section are identical in all situations studied because the only difference between the expansion and the rotation model in dynamic linear programming are in the crop enterprises. Hence, the logic and method of calculating the coefficients of the following enterprises applies to all situations studied.

c. Livestock enterprises. In Situations I and IV, only one enterprise for each of deferred-fed calves and a 2-litter hog system were included in each year's production possibilities. In Situations II, III and V, rotations of 1, 2, 3, 4 and 5 years of deferred-fed calves and 2-litter hog systems were included in the original matrices. However, as pointed out by Dr. Herman O. Hartley of the Department of Statistics of Iowa State College, the rotation form of livestock enterprises is unnecessary. Hence, only one 2-litter hog system and one deferred-fed calf enterprise each year will be described in this section. The necessary capital, labor, feed grain, forage feed etc. coefficients for each livestock enterprise in each year are shown in Table 12, p.62a. In addition to the data shown in Table 12, production of one 2-litter hog system in years 1, 2, 3, and 4 contributes $428.11 to the following year's capital supply. Production of one calf on deferred-feed adds $205.14 to the following year's capital supply. The net revenues (c_{jk}'s), of both livestock enterprises are discounted in years 2, 3, 4, and 5. The discounting of net revenues was described previously. As has already been explained in a previous section, the forage feed requirement (a_{ijk}), of both livestock enterprises is included in the year of production and also in the year subsequent to production in order that unused forage feed of one year may be transferred for use the following year.

d. Grain buying. The capital requirement coefficient (a_{ijk}), for grain buying in each year of each situation is \$1.43. The feed grain coefficient (a_{ijk}), is -1.0 in each year of each situation. One bushel of corn costs \$1.43 of capital and contributes one bushel of feed grain to the feed grain supply of that year. The intermediate capital coefficient in each year of each plan is -\$1.33. The net return in year 1 in each situation is \$-0.10 = \$1.33 - \$1.43. Because it is a minus, it is a cost. Thus, the purchase of 1 bushel of feed grain in year 1 of any situation subtracts \$0.10 from the net returns of that year; but, it adds \$1.33 to the amount of capital available in year 2. Again, the net returns from grain buying, even though the c_{jk}'s are negative, are discounted in years 2, 3, 4 and 5 in each situation.

e. Capital transfer. In order that unused capital of one year may be used the following year, a capital transfer activity is included in the original matrices of all situations for years 1, 2, 3, and 4. The net revenue (c_{jk}), of each capital transfer activity is equal to zero in the original matrices. The capital requirement coefficient in each year is +1.0. The intermediate capital transfer coefficient (a_{ijk}), is -1.0. Hence, in year 1, \$1.00 of unused capital may be transferred for use in year 2, if it is needed. Since year 5 is the last year of the plan, no capital transfer activity is included in year 5's activities.

f. Family living. The necessary a_{ijk} and c_{jk} coefficients for family living (household consumption), are described earlier in the appendix. In year 1 of Situation I, no charge was made for family living. Hence, the capital coefficient (a_{ijk}), equals zero. In all other years of all other situations, the capital coefficient (a_{ijk}), for family living equals +1.0.

3. Original matrices for Situations II, III, IV and V.

Situations II, III, IV and V employed the rotation model for obtaining their respective solutions. Only the coefficients for the cropping enterprises of the above situations will be discussed in this section.

The crop rotation opportunities for the above situations were described in the section on crops. Likewise, the necessary calculations and logic for computing the crop coefficients of the various rotations were discussed previously. To avoid repetition, only one example will be presented in this section on the computations of coefficients for a rotation.

Example: Consider the C_{11}-C_{21}-C_3-d_4-d_5 rotation on

Land A in Situation II. Since Situation II employed the rotation model, there are no intermediate land products. Thus, the only land requirement coefficient (a_{ijk}), necessary for this rotation is +1.0 opposite the Land A restriction (b_{11}), in year 1. The net revenue coefficient (c_j), for C_{11}-C_{21}-C_3-d_4-d_5 on Land A is obtained by adding the net revenue from C_{11} to the discounted net revenues of C_{21} and C_3. There are three capital requirement coefficients (a_{ijk}'): one for C_{11} in year 1, one for C_{21} in year 2 and, one for C_3 in year 3. The net revenues for C_{11}, C_{21}, and C_3 are calculated in the same way as described for C_{11} on Land A for Situation I. The capital requirement (a_{ijk}), for C_{11} equals cost of fertilizer plus constant cost per acre plus (variable cost per acre times yield of C_{11}). However, the capital requirement of C_{21} equals (-yield of C_{11} per acre times price of corn) plus cost of fertilizer plus constant cost per acre plus (variable cost per acre times yield of C_{21}). Likewise, the capital requirement (a_{ijk}), for C_3 equals (-yield of C_{21} per acre times price of corn) plus cost of fertilizer plus constant cost per acre plus (variable cost per acre times yield of C_3). Finally, the intermediate capital product (a_{ijk}), opposite year 4's capital restriction equals (-yield of C_3 x price of corn). There is an intermediate capital product because in years 4 and 5 the land is idle. The other coefficients for labor and feed grain for C_{11}, C_{21}, and C_3, in years 1, 2, and 3 respectively are obtained from Tables

10 and 11. The coefficients for feed grain have a minus sign as the production of C_{11}, C_{21}, and C_3 adds to years 1, 2, and 3's feed grain supply.

B. Appendix B. Note on the Solving of Dynamic Linear Programs on the IBM Type 650 Magnetic Drum Data Processing Machine

It is obvious that the size of the original matrix in dynamic linear programming necessitates the use of an electronic computer to solve the problem. For example, with corn, oats, hay (on Land A and Land B), family living, 2-litter hog system, deferred-fed calves, grain buying, family living and capital transfer as the only activities in year 1, the resulting original matrix for the expansion model for Situation I was 84 x 180. With the same production possibilities, but with fewer restrictions (due to the rotation model), an original matrix of 30 x 178 was obtained for Situation II. Because an IBM 650 was used in this study, the original matrices had to be set up in a form suitable for solution on the IBM 650.

The first step in preparing the matrix for solution on the IBM Type 650 Magnetic Drum Data Processing Machine is to code the data. A "library" program of instructions for the IBM 650 is available. The program will accomodate a 97 by infinity original matrix. The coding of the data consists of adjusting all numbers in the matrix (by multiplication or division), to as near 00001.00000 as possible. The net revenue or net price (c_{jk}), row does not have to be adjusted. The data in the matrices in this study were coded to as near 00001.00000 as possible. The P_0 columns were also coded. However, it is believed by the author, that rounding errors would have been smaller had it been possible to leave the original P_0 columns uncoded.

In coding data, any column or any row (including that row in the P_0 column), may be multiplied or divided by any constant. To decode the final answer, do the opposite to what was done to code the data. For example, if activity P_{30} is in the final plan, and the vector was multiplied by 100 to code it; to decode the answer, divide by 100. Or, if row 35 was divided by 10 to code it; multiply the figure in row 35 to decode the answer if restriction 35 (row 35), is in the final plan.

The time required to obtain a solution on the IBM 650, depends upon the number of restrictions much more than on the number of activities in the matrix. An 84 x 180 matrix required 165 iterations. The computing time required was approximately 46 hours. On the other hand, a 32 x 179 matrix required 78 iterations and 9 hours of computing time. Because of the large computing cost in using the expansion model in Situation I, Situations II, III, IV and V used the rotation model. The rotation model has fewer restrictions and, therefore, requires less time to obtain a solution. Hence, the cost was less by using the rotation model than by using the expansion model for the same production possibilities and non-land restrictions.